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A SIMULATION TECHNIQUE FOR ANALYZING EFFECT OF GPS RECEIVER CHARACTERISTICS ON PERFORMANCE OF A FIRE DIRECTION SYSTEM

P.T. Rojatkar and S.V. Gade

Armament Research and Development Establishment (ARDE), Pashan, Pune 411021, India

Requirement of improving accuracy at long ranges with a short reaction time is the main rationale behind any Fire Direction System (FDS). Conventionally a number of options are available to achieve this. Global Positioning System (GPS) based competent munition is a new addition to this family providing an inexpensive means of fire direction. In GPS based FDS, positional information received from the pilot projectile is used to improve accuracy of subsequent firings. This paper aims at analyzing effects of various GPS receiver characteristics on the accuracy of FDS. A mathematical model to workout extrapolation error, which is a measure of accuracy of the FDS, is presented in the paper. Effects of GPS receiver characteristics on extrapolation error have been analyzed for gun ammunition of 30-km range under influence of simulated MET conditions. A brief comparison between GPS based FDS and Fire Direction Radar (FDR) is also given in the paper.

INTRODUCTION

Conventional artillery weapon systems suffer two major shortcomings, viz. poor first round accuracy and slow response to call for fire. These drawbacks make them unsuitable for engaging today's mobile and fleeing targets. Guided projectiles and fire direction systems of different types are employed to overcome these shortcomings. The earlier option though ideal, is too expensive to be used on a mass scale, and thus, the later option is most widely used.

Introduction of Global Positioning System (GPS) has revolutionized all facets of military operations. Use of GPS in artillery projectiles i.e. competent munition, has provided an accurate and inexpensive means of fire direction. Fig. 1 shows operation of a competent munition. Here, an onboard GPS reccTver acquires positional data during the flight of the projectile, which is telemetered to a ground station. This projectile is programmed for self-destruction prior to impact (at 70 to 90% of the range) to preserve element of surprise. The impact point coordinates are then predicted by extrapolating the trajectory beyond the self-destruct (SD) point. The offset between the target and fictitious impact point

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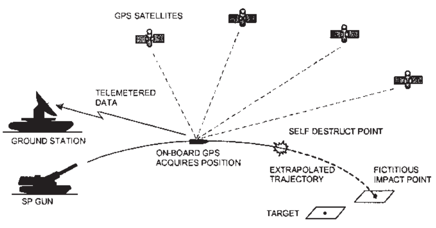

Figure 1: Operation of a GPS based fire direction system (FDS).

is used to calculate correction to the fire unit, to improve accuracy of subsequent projectiles. This accuracy depends on accuracy of prediction of pilot projectile's impact point. This in turn depends on accuracy of projectile tracking and trajectory extrapolation.

The distance between actual and fictitious impact point, which is termed as trajectory extrapolation error, is a measure of accuracy/ performance of the FDS. Mathematical model for calculation of extrapolation errors is presented in the paper. Similarly effects of various GPS receiver characteristics on the extrapolation error have been discussed. For the purpose of analysis, gun ammunition of 30-km range under the influence of simulated MET conditions is considered. A brief comparison between GPS based FDS (here after referred as only FDS) and FDR is also given in the paper.

MATHEMATICAL MODEL

As discussed earlier, performance of FDS depends on accuracy of determination of coordinates of fictitious impact point. This accuracy can be easily determined if these coordinates are compared with those of the actual impact point (in case the pilot projectile was allowed to impact on ground). The distance between these two points is termed as extrapolation error. Methodology for calculation of extrapolation error is discussed in this section.

Calculation of extrapolation error is a two step process. In the first step, the trajectory parameters (time, position and velocity) and the coordinates of actual impact point of the pilot projectile are obtained. In reality, the trajectory data is acquired from an onboard GPS receiver, whereas the coordinates of actual impact point are obtained by surveying the site of fall of shot. Since the paper deals only with the simulation technique, trajectory parameters and impact point coordinates are generated by trajectory computations. In these computations, use of simulated MET conditions is made in place of actual MET data. Trajectory of the projectile is computed using a 3-DOF point mass model. The equations of motion using point mass model are written as:

$$\ddot{x} = -G\,V\left[V_X - W_X\right] \tag{1}$$

$$\ddot{y} = -g(y) - G\,V\left[V_Y - W_Y\right] \tag{2}$$

$$\ddot{z} = -G\,V\left[V_Z - W_Z\right] \tag{3}$$

Where

$$G = \frac{\pi\,\rho(y)\,d^2 C_D V}{8 m_p} \tag{4}$$

V_X, V_Y, V_Z	– Projectile velocity components along X, Y, Z-axes of reference coordinate system respectively (m/sec).
V	– Resultant projectile velocity (m/sec).
W_X, W_Y, W_Z	– Wind velocities along X, Y, Z axes respectively (m/sec).
$\ddot{x},\ddot{y},\ddot{z}$	- Projectile accelerations along X, Y, Z axes respectively (m/sec^2).
$g(y)$	- Gravitational acceleration at altitude y (m/sec^2).
$\rho(y)$	- Density of air at altitude y (kg/m^3).
C_D	- Coefficient of drag.
m_p	- Mass of projectile (kg).
d	- Diameter of projectile (m).
x, y, z	- Suffixes for range, altitude and line respectively.

In the second step, coordinates of fictitious impact point are obtained. For this, trajectory data up to 70 to 90% of the range (i.e. up to the SD point) is retained. The remaining 30 to 10% of the trajectory is constructed by performing trajectory computations to obtain coordinates of fictitious impact point. These trajectory computations can be accurate, only if the weather (MET) conditions at the time of firing are accurately known. But this requirement contradicts with the basic aim of the FDS, i.e. independent operation of weapon system without use of a MET data acquisition system (to achieve fast response time to call for fire).

Non-availability of MET data for trajectory extrapolation is compensated by use of Trajectory Influence Parameters (TIPs). TIPs are parameters, which cause the trajectory to deviate from the standard. Methodology of obtaining the TIPs from the trajectory parameters of pilot projectile is discussed below.

Solution to equations of motion (1, 2 and 3) is obtained by using Runge-Kutta method. Fig. 2 shows a flowchart for solution of equation of motion in X-direction. This flow chart is modified to obtain TIPs. For this, displacement X in the standard atmosphere is first calculated. This displacement is compared with the displacement in the actual atmosphere (obtained from simulated trajectory data). If the difference between these two displacement values is more than the desired accuracy level, then the value of h (time interval used in Runge-Kutta method of solution) is changed by a small amount T_X and value of X

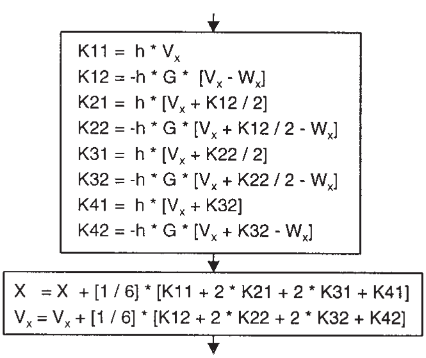

Figure 2: Solution of equation of motion in X direction.

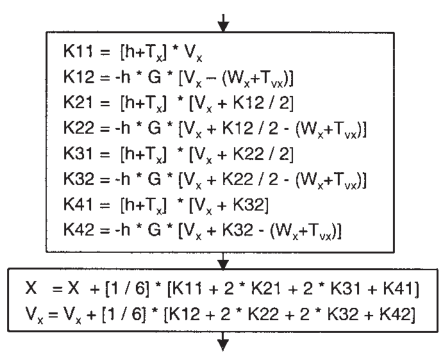

Figure 3: Flowchart for trajectory extrapolation with TIPs.

is re-computed. This process is repeated till the difference between the two displacement values converges to a desired accuracy level. After this, velocity V_X in the standard atmosphere is computed and compared with the velocity in the actual atmosphere. If the difference between them is more than the desired accuracy level, the value of W_X is changed by a small amount T_{VX} and value of V_X is recomputed. This process is repeated till the difference between two velocities converges to a desired accuracy level. Final values of T_X and T_{VX} for which, the calculated values of displacement and velocity match well with those in the actual atmospheric conditions are termed as TIPs. TIPs are calculated at different altitude levels using trajectory data in the ascending phase of the trajectory. Similar procedure is followed for obtaining TIPs in Z direction.

Procedure followed for calculation of TIPs in Y direction is slightly different. Here the velocity comparison is used to obtain (T_{VZ}) a correction factor for G (which takes care of weather conditions other than wind). Corrected value of G is then used in calculation of TIPs in X and Z direction. Procedure for obtaining Tz is same as for T_X.

The trajectory beyond SD point is extrapolated using TIPs (Fig. 3). For these computations, position and velocity at SD point becomes initial conditions. The values of h, wind and G are corrected using the corresponding TIPs. Extrapolation computations yield coordinates of fictitious impact point. Once the coordinates of actual and fictitious impact point are known, extrapolation error can be calculated.

GPS RECEIVER CHARACTERISTICS

FDS uses GPS receiver for time, position and velocity measurement. As true with any measurement system, these measurements are also not free from errors. The GPS output is updated at a regular time interval, reciprocal of which is termed as update rate. Once the pilot projectile is placed inside gun tube, onboard GPS receiver is masked from GPS satellite signals. After firing, finite time is required to establish the satellite lock on and resume the measurement. The time during which the GPS fails to provide any data after launch is known as Time to First Fix (TFF). The time, position, velocity, update rate and TFF are referred as GPS receiver characteristics. These characteristics influence the performance of FDS.

SIMULATED MET CONDITIONS

Simulated MET conditions help in generating different atmospheres, which can be present at the time of firing. MET simulation is used in place of actual MET data in the trajectory computations. MET data generally include measurement of wind (velocity and direction), air pressure, temperature and relative humidity at different altitude levels. The wind in reality is highly unpredictable and its velocity and directions can change in different layers of altitude. To account for this unpredictability, four types of wind patterns (Fig. 4) namely constant, sinusoidal, ramp and combination are considered for simulation. It is also assumed that, the projectile encounters only range wind and the crosswind is absent.

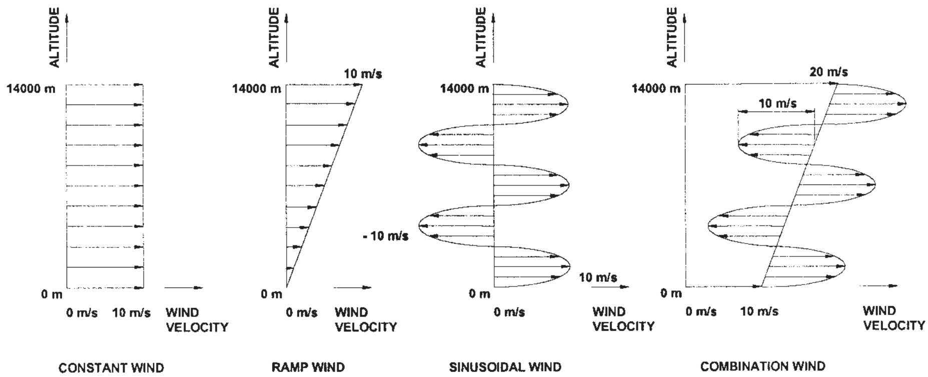

Figure 4: Wind patterns used for simulation (wind strength = 10 m/sec).

To simulate the pressure changes, it assumed that the air pressure at meteorological datum plane (MDP) is disturbed by 5%. Similarly, for temperature simulation, temperature at MDP is disturbed by 15°C. Both pressure and temperature disturbances at MDP, cause variations in air density at all altitudes above MDP. Temperature variations additionally lead to variations in the velocity of sound.

RESULTS

Effects of various GPS receiver characteristics on the extrapolation error are discussed in this section. These errors have been worked out for different wind patterns (wind strength of 10 m/s), pressure disturbance of 5% and temperature disturbance of 15°C. The performance comparison of GPS receivers based on C/A code and P code is also presented. For the purpose of analysis, a gun projectile having a range of 30-km in the standard atmosphere is considered.

Model validation

For validation of mathematical model, extrapolation errors for different self-destruct ranges are worked out. It is evident from Table 1 that maximum extrapolation error observed is only –4.02 m and hence the model is fairly accurate.

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Table 1: Extrapolation errors for different SD ranges

Extrapolation error (m) (Update rate = 4/sec: TFF = 0 sec: Velocity jump X and Y= 0 m/s: Displacement jump X and Y = 0 m)						
SD range (%)	Constant wind	Sinusoidal wind	Ramp wind	Combination wind	Pressure disturbance	Temperature disturbance
90 %	0.12	- 0.20	0.02	- 0.21	- 0.98	- 0.62
85 %	0.12	- 0.07	0.01	0.05	- 1.84	- 1.12
80 %	0.09	0.14	0.03	0.24	- 2.59	- 1.78
75 %	0.04	0.17	0.00	0.58	- 3.30	- 2.96
70 %	0.00	- 0.40	0.01	0.13	- 3.81	- 4.02

Time

GPS time measurement is based on atomic clock and is accurate to few nanoseconds. To study the effect of time measurement error, trajectory extrapolation computations (standard atmosphere, SD range =70%) were carried out with two values of time interval h (h = 0.1 sec and h = $0.1+500\times10^{-09}$ sec i.e. increased by time measurement error). It was observed that the ranges obtained for these two values of h differ only by 1.47×10^{-03} m. This signifies insignificant contribution of time measurement error towards extrapolation errors.

Positional accuracy

Positional accuracy of 15 m can be achieved by using differential GPS (DGPS) with C/A code whereas the P code gives sub-meter class accuracy. The errors introduced by the GPS satellites do not change continuously from one update to other. Over a time period equal to flight time of the projectile (about 90 sec), only one variation/ jump is expected. During the flight of the projectile this jump can occur anywhere in the flight path.

To study the effect of positional inaccuracy, a positional error of 15 m for C/A code and 1 m for P code, is deliberately introduced in the trajectory data. For this, trajectory data in X and Y direction is jogged by an amount equal to position error. As this jog/ jump occurs anywhere in the flight path, the extrapolation error will depend on the range at which the jump occurs. To account for this, the extrapolation errors are worked out for different jump locations. It can be seen from Table 2, that the C/A code based GPS receiver gives much larger extrapolation error compared to P code receiver.

Table 2: Effect of positional accuracy

Extrapolation error (m) (Update rate = 4/sec : TFF = 0 sec : SD Range =70%: Velocity Jump X and Y = 0 m/s)												
Jump location (% range)	Constant wind		Sinusoidal wind		Ramp wind		Combination wind		Pressure disturbance		Temperature disturbance	
	C/A Code	P Code	C/A Code	P Code	C/A Code	P Code	C/A Code	P Code	C/A Code	P Code	C/A Code	P Code
0 %	- 24.50	- 1.79	- 25.05	- 1.91	- 24.57	- 1.76	- 24.90	- 1.67	- 28.66	- 5.48	- 28.59	- 5.71
10 %	- 25.26	- 1.57	- 25.79	- 1.98	- 26.41	- 1.66	- 27.01	- 1.81	- 29.33	- 5.48	- 29.22	- 5.68
20 %	- 28.78	- 1.48	- 27.23	- 2.05	- 26.79	- 1.68	- 28.97	- 1.84	- 32.63	- 5.74	- 30.64	- 5.78
30 %	- 27.54	- 1.73	- 28.12	- 2.10	- 27.61	- 1.80	- 28.00	- 1.85	- 31.83	- 5.68	- 31.82	- 5.86
70 %	- 26.23	- 1.59	- 26.64	- 2.11	- 26.28	- 1.73	- 26.48	- 1.72	- 30.22	- 5.56	- 30.22	- 5.74

Accuracy of velocity

GPS based velocity measurement is accurate up to 0.5 m/s for C/A code and 0.1 m/s for P code. Methodology used for working out the extrapolation error is similar to that of positional accuracy. Extrapolation errors worked for two types of GPS reccivers (C/A code and P code) and for different jump locations are listed in Table 3. It can be seen that, velocity inaccuracy of 0.5 m/s (C/A code) has a significant effect on extrapolation error. P code receivers yield much lower extrapolation errors.

Table 3: Effect of accuracy of velocity

Extrapolation error (m) (Update rate = 4/sec : TFF = 0 sec : SD Range =70%: Displacement jump X and Y= 0 m)												
Jump location (% range)	Constant wind		Sinusoidal wind		Ramp wind		Combination wind		Pressure disturbance		Temperature disturbance	
	C/A Code	P Code	C/A Code	P Code	C/A Code	P Code	C/A Code	P Code	C/A Code	P Code	C/A Code	P Code
0 %	- 22.43	- 4.31	- 22.53	- 4.67	- 22.06	- 4.56	- 21.89	- 4.36	- 26.31	- 8.44	- 26.34	- 8.66
10 %	- 22.45	- 4.56	- 22.60	- 4.70	- 22.62	- 4.55	- 22.20	- 4.49	- 26.73	- 8.52	- 26.69	- 8.65
20 %	- 23.68	- 4.93	- 23.52	- 5.31	- 23.32	- 4.34	- 23.34	- 4.23	- 28.24	- 8.47	- 27.76	- 8.91
30 %	- 22.73	- 4.55	- 22.26	- 4.57	- 22.57	- 4.79	- 22.16	- 4.72	- 26.02	- 8.75	- 26.29	- 9.25
70 %	- 23.34	- 4.49	- 23.70	- 4.94	- 23.23	- 4.62	- 23.03	- 4.52	- 27.44	- 8.53	- 27.41	- 8.68

Update rate

The update rate is an important parameter of FDS, which should at-least match with the reciprocal of the time interval used for trajectory computations. GPS receivers have very slow update rate of 1–4 updates/sec. For obtaining TIPs, the tracked trajectory data at short time intervals (0.1 to 0.01 sec) is required. Interpolation of (Lagrange interpolation) GPS information accurately generates this data. This enables use of slow update rate for extrapolation without sacrificing accuracy, as evident from Table 4.

Table 4: Effect of update rate

Extrapolation error (m) (TFF =0 sec: SD Range =70%: Velocity jump X and Y = 0 m/s: Displacement jump X and Y= 0 m)						
Update rate (updates/sec)	Constant wind	Sinusoidal wind	Ramp wind	Combination wind	Pressure disturbance	Temperature disturbance
1	- 0.12	- 0.56	- 0.05	- 0.31	- 3.89	- 4.17
2	- 0.11	- 0.35	0.03	- 0.22	- 3.78	- 4.04
4	0.00	- 0.40	0.01	0.13	- 3.81	- 4.02

Time to First Fix (TFF)

TFF is a period during which GPS fails to provide any data. Thus, trajectory extrapolation in TFF region will have to be performed using standard atmospheric conditions. This would result in additional inaccuracies in estimating coordinates of point of impact. GPS receivers have TFF up to 10 seconds. As evident from Table 5, TTF (> 6 sec) is also a major contributor towards extrapolation errors.

Table 5: Effect of time to first fix (TFF)

Extrapolation error (m) (Update rate = 4/sec: SD Range =70%: Velocity jump X and Y = 0 m/s: Displacement jump X and Y = 0 m)						
TFF (sec)	1	2	4	6	8	10
Constant wind	0.18	0.69	2.43	4.94	7.94	11.29
Sinusoidal wind	- 0.52	- 0.21	- 0.90	- 0.10	- 1.83	0.25
Ramp wind	0.01	0.02	0.14	0.44	0.92	1.62
Combination wind	0.25	1.13	2.33	5.90	7.66	13.88
Pressure disturbance	- 3.80	- 3.79	- 3.57	- 3.02	- 2.11	- 0.86
Temperature disturbance	- 4.02	- 4.00	- 3.86	- 3.58	- 3.17	- 2.69

DISCUSSION AND CONCLUSIONS

Application of GPS in fire direction has generated a good alternative to conventional Radar based fire direction systems. Merits and demerits of both the systems are discussed here.

Accuracy of range measurement by FDR is usually of the order of 0.05% of range i.e. an error of 15 m at a distance of 30 km. This error becomes significant at long ranges. Positional accuracy of DGPS (C/A code) is also 15 m but is range independent. Use of FDS is therefore preferred for long range application.

For velocity measurement, the FDR accuracy is typically 0.02% of velocity (50 to 3000 m/s). Thus for muzzle velocity of 800 m/s, this error would be 0.16 m/s. GPS based velocity measurement is accurate up to 0.5 m/s for C/A code and 0.1 m/s for P code. Thus for velocity measurement, both P-code basod FDS and FDR are equally accurate. The FDR is however superior to FDS in respect of update rate and TFF.

The cost of FDR is very high compared to GPS based FDS (the cost ratio is almost 100). Higher cost renders FDR a good value target for the enemy. The FDR however, does not require any special pilot projectile as required by FDS. The cost of the competent munition is recurring in nature. Moreover, it also demands additional logistic resources.

The FDR has distinct disadvantage of its heavy active radiation. This makes detection of radar easy. Telemetry system fitted in the pilot projectile uses a small power transmitter. Even if this projectile is detected, it does not reveal the position of fire unit. The ground station of FDS is passive in nature and cannot be detected. This is a very significant advantage of FDS over FDR and this factor will largely dominate choice between FDR and FDS.

Today the main threat to the GPS receivers is from jamming and spoofing. The jamming and spoofing is easy since GPS signals are transmitted by the satellites at known frequencies and modulation characteristics and at a low signal to noise ratios. A compact, lightweight jammer planted on ground or attached to a balloon can effectively jam a large area. Typically a 1W jammer can effectively cover a range of 70 km.

To overcome the spoofing and jamming a lot of efforts are required to be put in. This will include use of antijam filters, nulling antenna, acquisition of Y code, low power atomic clocks, beam forming antennas, increased signal power, use of Lm signals, digital beam steering antennas etc. Unfortunately the GPS receivers available today do not have these facilities. The challenge is to build the GPS recelvers with anti-jam and anti spoofing facilities while keeping the size compact and cost to a low level.

The salient findings of this study are concluded below.

- Performance of FDS depends on GPS receiver characteristics like, accuracy of time, positional accuracy, accuracy of velocity, update rate and TFF.
- Use of TIPs in place of MET data for trajectory extrapolation yields accurate results.
- Extrapolation error largely depends on positional accuracy, accuracy of velocity and TFF. The accuracy of time and update rate have an insignificant effect.
- The pressure and temperature variations leads to larger extrapolation errors compared to different wind patterns. These errors however can be reduced, if values of pressure and temperature at MDP are accurately known.
- The GPS receivers based on C/A code, which has inferior accuracy (positional and velocity) compared to P code receivers, give large extrapolation errors.
- Security to GPS signals against jamming and spoofing is the main technological challenge of tomorrow.
- Performance of FDS based on a P code GPS receiver will be superior compared to conventional radar based fire direction systems. Also considering other advantages, the GPS based FDS has a potential of becoming a good alternative to FDR.