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ACCEPTANCE CRITERIA FOR FIRE PREDICTION ACCURACY

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This paper proposes a methodology to verify the indirect fire prediction accuracy of tube launched artillery with minimal additional costs and examines guidelines for acceptance criteria to assess fire prediction accuracy.

BACKGROUND

The process that precedes the ability to do accurate indirect artillery fire prediction usually involves a fairly extensive firing trial. The data from this firing trial is then reduced to MSL and standard conditions yielding calibration factors at discrete elevations per charge. Curves are then fitted to this experimental data in order to provide calibration data for intermediate QE's. At this stage the data can be implemented in a Fire Control Computer (FCC). Generation of range tables goes one step further before one can effectively predict. When doing indirect fire predictions using the range tables one can not expect to be bang on target, but one can expect, with a reasonable probability, to be close to the target. One of the reasons for this no-deterministic nature of fire predictions is to be found in the fact that the calibration curves on which the range tables are based are approximations of actual firing data. There are also other variations in e.g. muzzle velocity which will vary from the chosen muzzle velocity on which the prediction will be done. (See reference [1] for more detail)

Thus, the primary challenge is, once a new range table or set of calibration data for a fire control computer is introduced, how should the user define criteria on which to evaluate and validate the indirect fire prediction accuracy, especially in the face of some of the claims made by the suppliers of such systems.

VERIFICATION

Due to monetary constraints a comprehensive verification exercise after extensive range table firings is generally out of the question and even a limited verification firing is often ruled out. We decided to explore a cost-effective theoretical verification. The verification is done by using the original data from which the range table or calibration was derived. Application of the prediction process using the Mean Point of Impact (MPI) of the firing data as the designated target and applying the prediction process should reproduce the original QE and Azimuth within certain limits, i.e. the acceptance criteria.

Experimental Data

The data on which this exercise is based was obtained during a fairly extensive Range Table trial. The test range was a typical battle school environment and the survey accuracy and MET procedures etc. were typical of an operational environment, with little of the luxuries of a test range. The number of shots fired per serial generally varied between 3 and 5. This provides little statistical information but is in line with the generally accepted artillery practice of firing a minimum of 3 shots to establish a representative MPI. Included in the data are firing with 6 different charges and boattail projectiles and the two top charges with Basebleed projectiles.

In Table 1 the results of the range table prediction is compared to the as-fired data. The differences in azimuth and elevation are computed and expressed as a distance on the ground.

Table 1: Synopsis of unadjusted HE verification data

Range	Quadrant Elevation				Azimuth			
	As-fired (mils)	RT Prediction (mils)	Δ QE (mils)	Δ Rge (m)	As-fired (mils)	RT Prediction (mils)	Δ Brg (mils)	Δ Drift (m)
4471	255	258	3	45	688	684	-4	-18
7347	523	527	4	29	670	671	1	7
8185	610	638	28	118	776	778	2	16
8168	940	917	-23	-99	760	764	4	32
6541	1150	1147	-3	-33	700	695	-5	-32
5819	263	265	2	34	621	618	-3	-17
8710	460	469	9	100	819	823	4	34
10048	610	608	-2	-13	757	759	2	20
10479	840	785	-55	-220	784	784	0	0
8054	1154	1146	-8	-104	716	713	-3	-24
7498	258	262	4	80	673	674	1	7
10932	461	465	4	53	793	794	1	11
12073	550	551	1	11	356	355	-1	-12
13387	840	794	-46	-230	345	351	6	79
10196	1164	1164	0	0	679	672	-7	-70
9442	246	247	1	24	674	673	-1	-9
13385	450	455	5	79	434	431	-3	-39

Range	Quadrant Elevation				Azimuth			
	As-fired (mils)	RT Prediction (mils)	Δ QE (mils)	Δ Rge (m)	As-fired (mils)	RT Prediction (mils)	Δ Brg (mils)	Δ Drift (m)
15696	685	672	-13	-91	796	801	5	77
15541	960	977	17	172	769	765	-4	-61
13236	1132	1127	-5	-94	293	263	-30	-390
11668	224	227	3	90	362	360	-2	-23
17616	500	510	10	154	766	773	7	121
18953	630	636	6	64	957	951	-6	-112
20189	900	945	45	405	941	919	-22	-436
17011	1145	1155	10	250	661	656	-5	-84
16046	251	254	3	96	804	808	4	63
21918	500	511	11	198	926	922	-4	-86
24766	640	654	14	165	1146	1143	-3	-73
27333	880	855	-25	-175	1074	1061	-13	-349
23072	1150	1149	-1	-30	878	873	-5	-113
14022	247	253	6	210	484	484	0	0
19663	490	490	0	0	1008	1004	-4	-77
23394	690	712	22	207	964	963	-1	-23
24729	800	790	-10	-16	1143	1136	-7	-170
21187	1140	1145	5	140	869	864	-5	-104
18986	260	264	4	168	973	974	1	19
25236	425	427	2	58	1166	1173	7	173
31572	690	680	-10	-140	1240	1238	-2	-62
33121	900	854	-46	-322	1190	1186	-4	-130
29124	1140	1141	1	36	1076	1083	7	200

FACTORS INFLUENCING ACCEPTANCE CRITERIA

For a statistical treatment of the data, the inherent assumptions are that the distribution of the rounds around the MPI is independent of one another, as is the distribution of the MPI's. Secondly the distribution are gaussian and independent in the range and drift directions.

Dispersion

The characteristic dispersion of the weapon system needs to be taken into account in the definition of acceptance criteria. The armaments industry often suffer from the 'single gun battery syndrome'. As most of the development work is generally carried out using a single gun, we tend to lose sight that, except in some special cases, most artillery doctrines currently prefer a battery (usually a minimum of four guns) as a fire unit. This syndrome could result in the imposition of harsh criteria for what is essentially intended to be an area weapon. The drift dispersion is generally much smaller than range dispersion and cognisance should be taken of this fact lest one arrives at the ridiculous situation where the MPI is located slightly off the target centre (to the left or right) and being penalized although it might be well within the lethal radius of the projectile (see Fig. 1). When actual dispersion data is rather sketchy (typically only 3 to 5 rounds) one could probably use the Chi-squared statistics to try and adjust the standard deviations to a more acceptable sample of 20 rounds. The ideal would be to have a larger sample of shots, but in the absence of reliable experimental dispersion data we resorted to historical data.

The dispersion is used as a single value with no discrimination to the contributing factors such as muzzle velocity variation and meteorological variations. The muzzle velocity variation effect is hopefully accounted for to a large extent in using the average muzzle velocity together with the mean impact position. Any biases introduced by the meteorological reports due to staleness etc. has not been removed.

Measurement of fall of shot

Fall of shot was measured by three OP's and the impact coordiantes were calculated by means of triangulation. This length of the sides of the error triangles provided a measure of the accuracy of the observation. Generally the triangulation procedure used to derive the impact coordinates is based on finding the intersection of the gravity lines of the three sides of the triangle. However, if one of the internal angles exceeds 90°, the intersection lies outside the triangle next to the longest side. Although this makes mathematical sense, we consider it improbable that the observed fall of shot will physically be outside of the error triangle. Thus the fall of shot impact coordinates are determined as the center of the inscribed circle (Fig. 2).

$$R_{Inscribed} = \frac{\sqrt{s(s-a)(s-b)(s-c)}}{s} \qquad s = \frac{1}{2}(a+b+c) \tag{1}$$

with s the semiperimeter and a,b,c the lengths of the sides of the error triangle respectively.

This method has two advantages:

- The point of impact is always located within the error triangle and thus the error triangles maintain a physical meaning.
- The inscribed circle usually fits in the largest area of the triangle, with the origin corresponding to the center of gravity of the triangle and thus acts as some kind of weighting function to favour measurements that are closer together.

We assumed the probability that the actual point of impact will lie outside the circle circumscribing the error triangle is very small. Thus assuming a normal distrubution for measurement errors, the standard deviation for the measurement error was taken as $6\sigma = 2R_{Circumscribed}$. Based on the above assumption one can calculate a probability for the actual fall of shot lying within the radius of the inscribed circle. For the test data the probability of the true impact lying within the inscribed circle was generally better than 95%.

$$R_{Circumscribed} = \frac{abc}{4\sqrt{s(s-a)(s-b)(s-c)}} \quad (2)$$

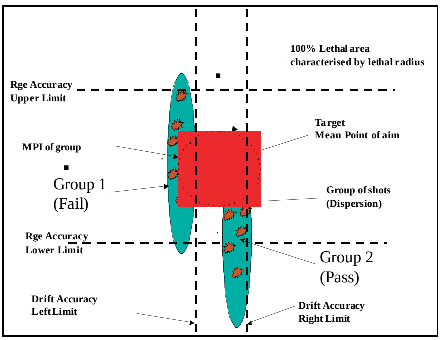

Fig. 1: Scenario for non-sensical fire prediction accuracy criteria.

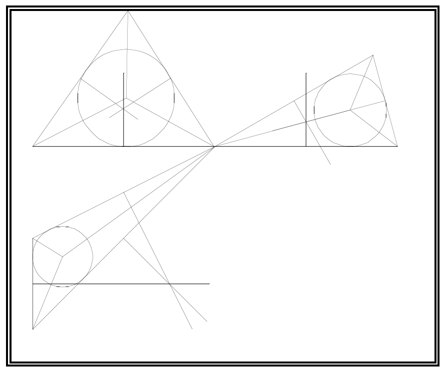

Fig. 2: Some typical scenarios for error triangles illustrating the relative derived impact positions.

Lethality

Lethality needs to be included in the derivation of fire prediction accuracy to establish some kind of minimum value beyond which arguments have academic rather than practical value. Lethality is characterized by the lethal area in m^2 for a shell detonated in a vertical orientation 1 m above the ground in an arena test. Lethal area and radius are defined in terms of anti-personnel targets (standing infantry in the open). The criteria for lethality is a fragment flux of 2 fragments/m^2 and perforation of at least 1.6 mm mild steel. The data in Table 2 provides some indication of this minimum cut-off distance, typically characterised by the lethal radius.

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Table 2: Theoretical lethality values as a minimum cut-off parameter in the definition of fire prediction acceptance criteria.

Shell Description	Explosive Type	Explosive Mass (kg)	Lethal Radius (m)	Lethal Area (m^2)
M107 HE	TNT	6.6	17.4	951
155 mm M1	0.3% HNS	8.5	29.9	2807
155mm V-LAP HE	0.3% HNS	4.5	18.8	1110
130 mm HE	0.3% HNS	3.9	23.8	1779
105 mm HE	0.3% HNS	2.7	18.4	1063

Adjusted Aimpoint Offsets

In the computations that follow, the deviations from the aimpoint have been adjusted to account for the lethal radius of the 155 mm M1 HE projectile (29.9 m) and for the measurement error as characterised by the radius of the inscribed circle. A fairly decent normal distribution for the offset in both range and drift between the predicted and actual aimpoints were obtained (see Fig. 3).

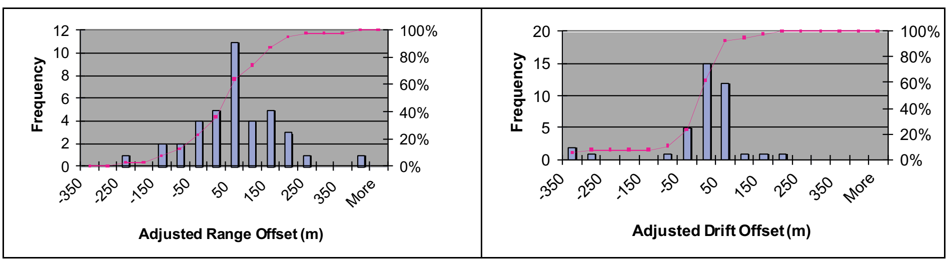

Figure 3: Distribution of the range and drift deviations relative to the point of aim after adjustments for the lethal radius and measurement uncertainties.

ACCEPTANCE CRITERIA

There are several options that come to mind for use as acceptance criteria. The use of independent dispersion related criteria, an ECEP-value or even a Hit Probability could be used. Reference [1] gives a good exposition in the fundamentals. Fire prediction accuracy is a measure of weapon system performance and as such one of the primary governing parameters is precision, consisting of consistency and accuracy.

Independent Dispersion related Criteria

The most obvious and intuitive criteria is probably analogous to the dispersion. This type of criterion would in general consist of two independent criteria, one for range and one for drift. Typical descriptive statistical data derived from the example is provided in Table 3. A simple graphical analysis shows that in the majority of cases, the predicted and actual aimpoints differ by less than 2PE (see Fig. 4).

Table 3: Descriptive statistics of the offsets

	Mean Offset	Std Deviation
Range	25.3	119.8
Drift	-32.9	109.2

One could indulge in some statistics to compare the actual offset values using the mean values of the standard deviation of the offsets as the population standard deviation. A drawback of this approach is that it does not account explicitly for the dispersion within a salvo. To rectify this deficiency, we examine the ECEP and Hit Probability concepts.

Equivalent Circular Probable Error (ECEP)

The equivalent circular probable error (ECEP) is also a useful single-valued parameter for defining acceptable limits. The ECEP definition given below accounts for the effects of miss distance but also includes the effects of dispersion.[2] An ECEP-value of 92 m result from this treatment on the data of Table 1.

$$R_{ECEP} = \sqrt{\sigma_T} \cdot \left[1 - \frac{\nu}{9\mu^2}\right]^{\frac{3}{2}} \tag{3}$$

$$\mu = 1 + \frac{\bar{x}^2 + \bar{y}^2}{\sigma_T^2} \quad , \quad \sigma_T = \sqrt{\sigma_R^2 + \sigma_D^2}$$
$$\nu = \frac{2\left(\sigma_R^4 + \sigma_D^4\right)}{\sigma_T^2} + \frac{4\left(\sigma_R^2 \bar{x}^2 + \sigma_D^2 \bar{y}^2\right)}{\sigma_T^2} \tag{4}$$

First Round Single Shot Hit Probability

For interest sake we also examined the data in terms of a first round hit probability on an arbitrary square target with dimensions of 100 m x 100 m. The first round single shot hit probability on a rectangular target with sides *2a* and *2b* respectively and with the the predicted point of impact offset relative to the intended point of aim by the coordinate pair *(μ,ν)* is given according to [2] by

$$P_h = \frac{1}{2\pi} \int_{(-a-\mu)/\sigma_x}^{(a-\mu)/\sigma_x} \int_{(-b-\nu)/\sigma_y}^{(b-\nu)/\sigma_y} e^{-(x^2+y^2)/2} dx.dy \tag{5}$$

Using the data of Table 1 a mean first round hit probability of 40% results on a 100 m x 100 m target.

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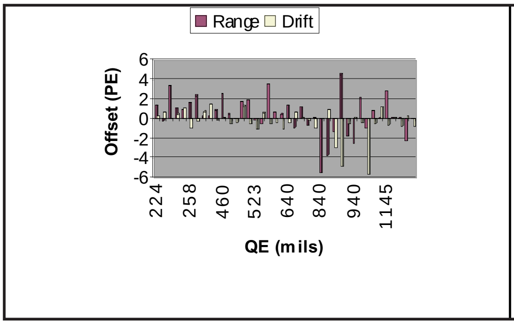

Fig. 4: Range and drift offsets of the predicted impact relative to the aimpoint as function of elevation.

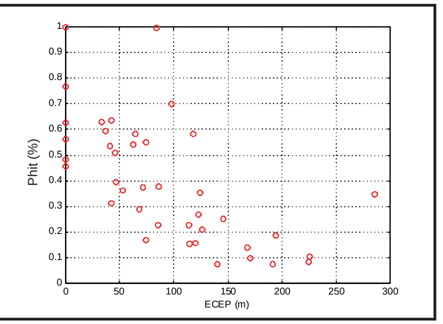

Fig. 5: Plot of the first round single shot hit porbability versus the ECEP-values.

CONCLUSIONS

From Fig. 4 one can deduce that the substantial deviations in drift occur around the elevations for maximum range and most probably for high angled fire just beyond the elevation for maximum range. This is generally a region where strong non-linearities exist. Firing at maximum ranges is an artificial imposition for range table trial purposes and in practice one would opt for the next higher charge to achieve the maximum range of the previous charge at more well behaved elevations. The larger variation experienced in the drift acceptance was somewhat unexpected. It could in part be attributed to the human factor, i.e. inaccuracies in the survey, orientation and laying of the gun. It is our experience that guns with an autolaying system generally perform much better with respect to deviations from line. This gun was however manually layed. Thus the degree of automation could also influence the final acceptance figures.

Thus one could possibly opt for a fire prediction accuracy that is

– within 2PE in both range and drift, or

– within an ECEP of 92 m which translates to 0.6% of the mean range, or

– a mean single shot first round hit probability of around 40% on a 100 m x 100 m target.

The values quoted here are indicative of what can be construed and one should be careful with regard to the indiscriminate application of these criteria and be critical of the interpretation thereof.

REFERENCES

1. Textbook of Ballistics and Gunnery. Volume 1, Her Majesty's Stationary Office, 1987
2. Engineering Design Handbook. Army Weapon Systems Analysis Handbook, Part One. DARCOM Pamphlet No. 706-101, November 1977