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THE INFLUENCE OF A PROJECTILE STABILITY SUBJECTED TO SOME CONTROL IMPULSE MOMENTS

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The control impulse moments are often used to adjust the trajectory for these precision munitions and they will affect the flight stability for these munitions. The paper analyses this kind of influence and a condition of flight stability to be satisfied when these impulse moments act on the projectile has been set up. It is useful for consideration of designing these precision munitions.

INTRODUCTION

Lately, the emphasis of development in munition technique is precision munitions, the trajectories of which need to be corrected. For a traditional projectile or rocket, the initial shot conditions (velocity, angle of fire, etc.) can be determinated by the position of a cannon and the observed target and their trajectories of flight can not be controlled or corrected after firing. In fact, there are various random disturbances which cause the deviation of the flight trajectory of a projectile or rocket relative to the expected trajectory or target. For precision munitions, the errors of the flight can be corrected, which is very important difference about the precision munitions and the traditional munitions.

The correcting forces or impulse moments are often used in the correction of trajectory. It is thoretically made clear that the flight stability of a projectile to be stabilized originally is influenced by the correcting forces or impulse moments. In reference[1], the effects of correcting forces are discussed in detail, but the correcting impulse moments that are often used are not concerned. In the paper, the case of the impulse moments that are applied on the spin-stabilized projectile to control the flight trajectory is analyzed.

COORDINATES

Coordinates on the ground $o-xyz$

The origin of coordinates is the center of the gun muzzle and Λ stands for the earth latitude there. The axis *ox* is on the horizontal plane and α_N stands for the included angle between the axis ox and the north, and the axis oy is upward vertically, and the axis oz is determinated by the right hand law.

Coordinates of velocity, $c - x_2 y_2 z_2$

The origin of coordinates is the center of mass (C.M) of projectile, the axis cx_2 is in line with the vector of velocity of the C.M. of projectile. θ and ψ are used to stand for the zimuth of the coordinate system relative to $o - xyz$, that is, $o - xyz$ is first taken to move parallelly to the C.M. of projectile and the new coordinate system $c - x'y'z'$ is obtained, then $c - x'y'z'$ is rotated about are axis cz' by θ where the axis cy_2 is obtained, and finally is rotated about the axis cy_2 by ψ, and then $c - x_2 y_2 z_2$ are obtained.

Coordinates of the projectile axis, $c - \xi\eta\xi$

The axis $c\xi$ is in line with the projectile axis and the direction pointed to the projectile nose is defined as the positive direction of $c\xi$. The azimuth of the coordinate system relative to $c - x'y'z'$ in defined as φ_1 and φ_2. The $c - x'y'z'$ in first rotated about cz' by φ_1 and then rotated about $c\eta$ by φ_2 where $c\eta$ is obtained by the $c\eta'$ rotating, and the final position of $c - x'y'z'$ rotating is $c - \xi\eta\xi$.

The attitude angles of the coordinate systems (δ_1, δ_2), (φ_1,φ_2), (θ, ψ) have the relationship as follows:

$$\delta_1 = \sin^{-1}\left[\frac{\xi_{y_2}}{\sqrt{\xi_{x_2}^2 + \xi_{y_2}^2}}\right] \tag{1}$$

$$\delta_2 = \sin^{-1}(\xi_{z_2})$$

$$\left(-\frac{\pi}{2} < \delta_1 < \frac{\pi}{2}, -\frac{\pi}{2} < \delta_2 < \frac{\pi}{2}\right)$$

Where

$$\begin{Bmatrix} \xi_{x_2} \\ \xi_{y_2} \\ \xi_{z_2} \end{Bmatrix} = \begin{bmatrix} \cos\psi\cos\theta & \cos\psi\sin\theta & \sin\psi \\ -\sin\theta & \cos\theta & 0 \\ -\sin\psi\cos\theta & -\sin\psi\sin\theta & \cos\psi \end{bmatrix} \begin{bmatrix} \cos\varphi_2\cos\varphi_1 \\ \cos\varphi_2\sin\varphi_1 \\ \sin\varphi_2 \end{bmatrix} \tag{2}$$

GENERAL EQUATIONS OF MOTION FOR PROJECTILES

Based on the coordinate systems above, the 6-D equations of motion for spin-stabilized projectiles can be created in the form as follows:

$$
\begin{cases}
\dfrac{\mathrm{d}v}{\mathrm{d}t} = \dfrac{1}{m} F_{x_2} \\
\dfrac{\mathrm{d}\theta}{\mathrm{d}t} = \dfrac{1}{mv\cos\psi} F_{y_2} \\
\dfrac{\mathrm{d}\psi}{\mathrm{d}t} = \dfrac{1}{mv} F_{z_2} \\
\dfrac{\mathrm{d}x}{\mathrm{d}t} = v\cos\psi\cos\theta \\
\dfrac{\mathrm{d}y}{\mathrm{d}t} = v\cos\psi\sin\theta \\
\dfrac{\mathrm{d}z}{\mathrm{d}t} = v\sin\psi \\
\dfrac{\mathrm{d}\omega_\xi}{\mathrm{d}t} = \dfrac{1}{C} M_\xi \\
\dfrac{\mathrm{d}\omega_\eta}{\mathrm{d}t} = \dfrac{1}{A} M_\eta - \dfrac{C}{A}\omega_\xi\omega_\zeta + \omega_\zeta^2 \operatorname{tg}\varphi_2 \\
\dfrac{\mathrm{d}\omega_\zeta}{\mathrm{d}t} = \dfrac{1}{A} M_\zeta + \dfrac{C}{A}\omega_\xi\omega_\eta + \omega_\zeta^2\omega_\eta \operatorname{tg}\varphi_2 \\
\dfrac{\mathrm{d}\varphi_1}{\mathrm{d}t} = \dfrac{1}{\cos\varphi_2}\omega_\zeta \\
\dfrac{\mathrm{d}\varphi_2}{\mathrm{d}t} = -\omega_\eta \\
\dfrac{\mathrm{d}\dot{\gamma}}{\mathrm{d}t} = \omega_\xi - \omega_\zeta \operatorname{tg}\varphi_2
\end{cases}
\tag{3}
$$

Where (F_{x2}, F_{y2}, F_{z2})are the projection parts of the resultant force $\overline{F}$ acted on the projectile in the coordinate system $c - x_2y_2z_2$; (M_ξ, M_η, M_ζ) are the projection parts of the resultant moment $\overline{M}$ about the C.M. of projectile in the coordinate system $c - \xi\eta\zeta$; (ω_ξ,ω_η,ω_ζ) are the projection parts of the total angle velocity $\overline{\omega}$ of a projectile in the coordinate system $c - \xi\eta\zeta$.

EFFECTS OF IMPULSE MOMENTS ON THE FLIGHT STABILITY

It is common means in the correction techniques of flight trajectory that the impulse moments are used to correct the ballistic errors. In general, the active time of the impulse moments is very short, mostly the order of milliseconds, and then the impulse moment is considered to complete instantaneously, which is equivalent to a great disturbance applied to a projectile during flying. Therefore, the effect of impulse on the flight trajectory and stability of a projectile can be analyzed in terms of the initial disturbance.

To assume a impulse in $\hat{P}$, the corresponding impulsive force is

$$P(t) = \hat{P}\delta(t - t_p) \tag{4}$$

Where $\delta(t)$ is the Direct function, t_p is the instant of impulsive action. The correcting forces and moments caused by the impulsive force are projected on the coordinates of velocity and on the coordinates of projectile axis, and then the impulse parts ($\hat{F}_{px2}$, $\hat{F}_{py2}$, $\hat{F}_{pz2}$) along with impulse moment parts ($\hat{M}_{px2}$, $\hat{M}_{py2}$, $\hat{M}_{pz2}$) can be obtained. According to the equations of motion (3), the increments of ballistic elements caused by the impulse $\hat{P}$ can be derived, that is

$$\begin{cases} \Delta v = \dfrac{\hat{F}_{px2}}{m}, \Delta\theta = \dfrac{\hat{F}_{py2}}{mv\cos\psi}, \Delta\psi = \dfrac{\hat{F}_{pz2}}{mv} \\ \Delta\omega_{\xi} = \dfrac{\hat{M}_{px2}}{C}, \Delta\omega_{\eta} = \dfrac{\hat{M}_{py2}}{A}, \Delta\omega_{\zeta} = \dfrac{\hat{M}_{p\zeta}}{A} \end{cases} \tag{5}$$

Some of the above increments of elements, such as Δv, Δθ, Δψ, etc., will influence the flight trajectory which are expected in the correction of trajectory. But some others may influence the flight stability of projectile, the following are discussion of the effects on stability.

In the formulation of the element increments (5), it can be approximately obtained that

$$\begin{cases} \partial\dot{\delta}_1 = \dfrac{\hat{M}_{p\zeta}}{A}, \partial\dot{\delta}_2 = \dfrac{\hat{M}_{p\eta}}{A} \\ \partial\delta_1 = \partial\delta_2 = 0 \end{cases} \tag{6}$$

Where

$$\partial\dot{\delta} = \sqrt{\partial\dot{\delta}_1^2 + \partial\dot{\delta}_2^2} = \frac{1}{A}\sqrt{\hat{M}_{p\eta}^2 + \hat{M}_{p\zeta}^2} \tag{7}$$

Eq. (7) is the change of angle velocity of attack caused by the impulse moment.

According to the theory of exterior ballistics, the differentiate equation of angle motion of attack under the simplify conditions is

$$\ddot{\Delta} - i2\alpha v\dot{\Delta} - k_z v^2 \Delta = 0 \tag{8}$$

Under the initial conditions, $\Delta_0 = 0$; $\dot{\Delta}_0 = \dot{\delta}_0 e^{i\nu_0}$, the solution of Eq. (9) is

$$\begin{cases} \delta = \dfrac{\dot{\delta}_0}{\alpha^*\sqrt{\sigma}} \sin\alpha^*\sqrt{\sigma}t \\ \nu = \nu_0 + \alpha^* t \end{cases} \tag{9}$$

Where

$$\alpha^* = \alpha v = \frac{C\dot{\gamma}}{2A}$$

$$k_z^* = k_z v^2$$

$$\sigma = 1 - \frac{k_z}{\alpha^2} = 1 - \frac{k_z^*}{\alpha^{*2}} \tag{10}$$

From the relations above, if a projectile has originally spin-stability ($\sigma > 0$), the condition of its spin-stability is not damaged by the impulse moment (Exceptionally, if the impulse part $\hat{F}_{px2}$ is very large and the increment Δv is also very large, the effect of Δv on the k_z/α^2 in σ should be analyzed, but this effect is generally small.

From the Eq. (10), the impulse moment causes the increase of the magnitude of angle of attack that is

$$\partial\delta_m = \frac{\dot{\delta}_0}{\alpha^*\sqrt{\sigma}} = \frac{\partial\dot{\delta}_0}{\alpha^*\sqrt{\sigma}} = \frac{\sqrt{\hat{M}_{p\eta}^2 + \hat{M}_{p\varsigma}^2}}{A\alpha^*\sqrt{\sigma}} \tag{11}$$

We consider that the increase of the magnitude of this angle caused by the impulsive moment must be smaller than some allowed limit, otherwise it is possible that the angle of attack become very large and damage the flight stability of projectile. To assume the allowed up limit of the increase of the this angle magnitude is $\partial\delta_{m0}$ then

$$\partial\delta_m \le \partial\delta_{m0}$$

hence

$$\sqrt{\hat{M}_{p\gamma}^2 + \hat{M}_{p\varsigma}^2} \le A\alpha^*\sqrt{\sigma}\,\partial\delta_m \tag{12}$$

Eq. (12) is very important and expresses obviously that if the flight stability of projectile after correcting is ensured, the square sum of the two parts of the impulse moment can not be excessively great in the process of trajectory correction. Eq. (11) is considered as a new condition of flight stability required in the action of impulse moment.

Eq. (12) can be changed into another form,

$$\sqrt{\hat{M}_{p\eta}^2 + \hat{M}_{p\varsigma}^2} \le \frac{C\dot{\gamma}}{2}\sqrt{1 - \frac{1}{S_g}}\,\partial\delta_{m0} \tag{13}$$

Where $S_g = \alpha^2 / k_z$, is the factor of spin-stability. From the Eq. (12), it can be shown apparently that for a given limit $\partial\delta_{m0}$, if a projectile has higher roll speed, or more stubby body, or stronger spin-stability, the impulse moment allowed by flight stability can be greater a little, in other words, the ability of projectile against the stronger disturbance of the impulsive moment is greater; contrarily, the ability is lower a little.

CONCLUSION

The effects of the impulse moment commonly used in the technique of the ballistic correction on the flight stability of a projectile are analyzed and a new condition of the flight stability under the action of impulse moment on the corrected trajectory is obtained. These are worthy for the technique of the ballistic correction currently being developed, especially for the choice of projects in design of ballistic correction units.

REFERENCE

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2. Fa Pu. 1980. "Exterior Ballistics", *National Industry of Defence Press*, China, 1980