TWO-PHASE FLOW MODEL OF GUN INTERIOR BALLISTICS

D. Micković¹, S. Jaramaz¹

¹ Faculty of Mechanical Engineering, 27.marta, 11000 Belgrade, Yugoslavia

The model of two-phase flow of solid granular propellant and its products of combustion in the gun barrel during interior ballistic cycle of ammunition whose propellant charge is ignited by the igniter is given. The theoretical model includes the balance equations of mass, momentum and energy for both phases, as well as necessary constitutive laws. The igniter efflux in the propellant chamber is obtained by incorporation in the model the two-phase flow model of igniter function. Convergent, unconditionally stable, numerical procedure is formed to solve the system of equations of the theoretical model. An original procedure of numerical grid combined adaptation to the flow field increase, caused by the projectile motion down the gun bore, is developed. The TWOPIB code for the computation of whole interior-ballistic cycle is developed. Verification of the model by the comparison with experimental data for the firings in the 100 mm gun is carried out. Computational results for some important parameters which can’t be measured are presented. The presented model enables more successful solutions of many interior ballistic problems.

INTRODUCTION

An exceptional practical importance of ignition and flamespreading through propellant charges caused the necessity of two-phase interior ballistic flow modelling. In the paper basic equations of two-phase flow model of complete interior ballistic cycle with real energetic impulse of the igniter are given. Detailed presentation of the two-phase flow model in the igniter and two-phase flow in the gun barrel during firing can be find in [1].

THEORETICAL MODEL

The theoretical modelling of processes in the propellant charge during ignition by the igniter is based on modelling of nonsteady reactive two-phase flow of unburned propellant grains and their gaseous combustion products. Because of great complexity of inves-
tigated physical phenomena certain assumptions usual in interior ballistic two-phase flow modelling are accepted [2]. From general conservation laws we pass to macroscopic balance equations by formal averaging over control volumes. Combined averaging procedure of Celmins and Gough is used.

The **basic equations** of two-phase flow in propellant chamber and gun barrel are:

Gas-phase mass-conservation equation

\[
\frac{\partial (\varepsilon \rho_g \vec{V}_g)}{\partial t} + \nabla (\varepsilon \rho_g \vec{V}_g) = \frac{\bar{m}}{W_{CV}} + \frac{\bar{m}_{g,fi}}{W_{CV}}
\]  

(1)

Particle-phase mass-conservation equation

\[
\frac{\partial R_p}{\partial t} + \langle R_p \vec{V}_p \rangle = -\frac{1}{\rho_p} \bar{m}
\]  

(2)

where \(\varepsilon, R_p\) are gas and particles volumetric fraction, \(\rho_g, \rho_p\) – gas and propellant density, \(\vec{V}_g, \vec{V}_p\) – velocity of gas and solid, \(\bar{m}\) – rate of interphase mass transfer, \(\bar{m}_{g,fi}\) – mass flow rate of gases through igniter side holes and \(W_{CV}\) is control volume. Single overbar denotes time average; double overbar denotes time average normalised by \(\varepsilon\).

Gas-phase momentum-conservation equation

\[
\frac{\partial (\varepsilon \rho_g \vec{V}_g \vec{V}_g)}{\partial t} + \nabla (\varepsilon \rho_g \vec{V}_g \vec{V}_g) = -\varepsilon \vec{V}_g + \frac{\bar{P}}{W_{CV}} \langle \vec{F} \rangle^i + \frac{\bar{m}}{W_{CV}} \vec{V}_g + \frac{\bar{m}_{g,fi}}{W_{CV}} \vec{V}_g
\]

(3)

Particle-phase momentum-conservation equation

\[
\frac{\partial (R_p \rho_p \vec{V}_p \vec{V}_p)}{\partial t} + \nabla (R_p \rho_p \vec{V}_p \vec{V}_p) = -R_p \vec{V}_p + R_p \vec{V}_p - R_p \vec{V}_p + R_p \frac{\bar{F}}{W_{CV}} \langle \vec{F} \rangle^i - \frac{\bar{m}}{W_{CV}} \vec{V}_p
\]

(4)

where \(p\) is pressure, \(P_p\) is intergranular stress and \(\vec{F}\) is interphase friction force. Interphase surface average is denoted by \(\langle \cdot \rangle^i\).

Gas-phase energy-conservation equation

\[
\frac{\partial \left(\varepsilon \rho_g \vec{H}_g - p\right)}{\partial t} + \nabla (\varepsilon \rho_g \vec{H}_g - p) = -R_p \frac{\bar{S}_{p}}{W_{p}} \langle q \rangle^i + \bar{\vec{V}}_p R_p \frac{\bar{F}}{W_{p}} \langle \vec{F} \rangle^i +
\]

(5)

\[+
\frac{\bar{m}}{W_{CV}} \left(\vec{H}_p + h_{c,p} + \frac{\bar{\vec{V}}_p \bar{\vec{V}}_p}{2}\right) + \frac{\bar{m}_{g,fi}}{W_{CV}} \vec{H}_g,fi
\]

Gas-phase energy-conservation equation

\[
\frac{\partial \left[R_p \rho_p \vec{H}_p - p - \vec{P}_p\right]}{\partial t} + \nabla \left[R_p \rho_p \vec{V}_p \vec{H}_p\right] = R_p \frac{\bar{S}_p}{W_p} \langle q \rangle^i -
\]

(6)

\[ - \bar{\vec{V}}_g R_p \frac{\bar{S}_p}{W_p} \langle \vec{F} \rangle^i - \frac{\bar{m}}{W_{CV}} \left(\vec{H}_p + h_{c,p} + \frac{\bar{\vec{V}}_p \bar{\vec{V}}_p}{2}\right)
\]

where \(\vec{H}_g, \vec{H}_p\) are gas and particle total enthalpy, \(\vec{H}_g,fi\) is total enthalpy of gases coming from the igniter side holes, \(h_{c,p}\) is heat of combustion of propellant and \(q\) is interphase heat transfer.
Additional determinations for some terms in previous equations are done by constitutive laws.

**Equation of state** for the gas phase is the Abel-Nobel relation. The statement of a constant density for the propellant grains represents the equation of state for the solid phase.

Supposing direct correlation with the pressure drop in the granular bed interphase friction force for particles of any shape is \( \langle F \rangle = \rho_g \| \dot{v}_g - \dot{v}_p \| | \dot{v}_p | f / \beta \). Interphase friction coefficient \( f \) is obtained from Ergun, Kuo-Nydegger or Wilcox-Krier law for non-fluidised bed, and from Anderssen law for fluidised bed [3].

Isotropic intergranular stress depending only on particles volumetric fraction is adopted. Gough-Zwarts, SNPE and Kuo-Summerfield intergranular stress laws are incorporated in the model [4].

The criterion for propellant ignition assumes ignition to have occurred when the surface temperature of a particle reaches a critical value:

\[
\bar{m} = 0 \quad \bar{T}_{ps} < T_{ign} \quad ; \quad \bar{m} > 0 \quad \bar{T}_{ps} = T_{ign}
\] (7)

The empirical pressure dependent relation \( \langle u \rangle = a \bar{p}^{n} + b \) is used for the propellant burning law (\( a, b \) and \( n \) are constants). The production rate of gases in the control volume is \( \bar{m} = R_p W_{CV} \rho_p \langle u \rangle \| S_p \| / W_p \).

Propellant surface temperature \( T_{ps} \) is defined from the energy conservation equation for the interphase control volume:

\[
h_g A_{ps} \left( \bar{T}_g - \bar{T}_{ps} \right) + \bar{m} C_p \bar{T}_p + \bar{m} h_{c.p} = h_p A_{ps} \left( \bar{T}_{ps} - \bar{T}_p \right) + \bar{m} C_p \bar{T}_{ps}
\] (8)

where \( A_{ps}, C_p \) and \( T_p \) are surface, specific heat and bulk temperature of propellant. Interphase heat-transfer coefficient \( h_g \) is obtained from convective heat transfer relations for granular beds: Gelperin-Einstein relation for non-fluidised beds and Butler-Lembeck-Krier relation for fluidised granular propellant beds. Particle-side heat-transfer coefficient \( h_p \) is obtained from thermal wave penetration depth considerations.

The “shadow” method of Spalding is used for the particle size calculation. The particle volume is: \( W_p = W_{po} (R_p / R_p^*) \) \( (R_p^* = “shadow” \ \text{particle volume fraction}; W_{po} = \text{initial particle volume}) \). The particle surface is given by the propellant form function.

Mass flow rate of black powder gaseous combustion products through igniter side holes is defined from the complete two-phase flow model of igniter real function in the propellant charge [1]. The igniter efflux is governed by the ratio of pressure in the propellant charge and corresponding pressure in the igniter \( p_{ig} \) (Table 1).
Table 1

<table>
<thead>
<tr>
<th>( \frac{p}{p_{ig}} \leq \left( \frac{2}{\kappa_{BP} + 1} \right)^{\frac{1}{\kappa_{BP}}} )</th>
<th>( \frac{p}{p_{ig}} &gt; \left( \frac{2}{\kappa_{BP} + 1} \right)^{\frac{1}{\kappa_{BP}}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{m}<em>{g,fi} = C_D \left[ 1 - R</em>{BP} \right] A_{fi} \sqrt{ \frac{p_{ig} \rho_{g,BP}}{1 + \frac{\rho_{BP} M_{fi}^2}{2}}} )</td>
<td>( \bar{m}<em>{g,fi} = C_D \left[ 1 - R</em>{BP} \right] A_{fi} \sqrt{ \frac{2 \kappa_{BP} p_{ig}}{\kappa_{BP} - 1} \rho_{g,BP} \left( \frac{p}{p_{ig}} \right) \left( \frac{\rho_{BP}}{\rho_{ig}} \right) \left( \frac{\kappa_{BP} + 1}{\kappa_{BP}} \right) } )</td>
</tr>
</tbody>
</table>

Remarks: \( A_{fi} \) - igniter side holes surface; \( \kappa \) - specific heat ratio of gas; subscript BP denotes black powder.

Discharge coefficient \( C_D \) and hole Mach number \( M_{fi} \) are determined in accordance with Sneck considerations of the flow in the gun-tube side holes.

The equation of projectile movement is:

\[
m_{pr} \frac{dV_{pr}}{dt} = (p + \rho_p) S_t - p_{fi} S_t
\]

where \( m_{pr} \), \( V_{pr} \) are projectile mass and velocity, \( p_{fi} \) is projectile frictional pressure \( S_t \) is gun-tube cross-sectional area. The projectile movement begins when a. sum of pressure and intergranular stress at the projectile base becomes greater than the projectile start pressure \( P_0 \). The projectile displacement is: \( dX_{pr} = V_{pr} \, dt \).

The finite-difference equations are derived by integrating of partial differential equations over the finite control volumes. For equations discretisation a conventional equidistant staggered grid is used with velocity nodes at the boundaries. An original strategy of combined numerical grid adaptation to the flow field increase, caused by the projectile motion, is developed. For the \( n \)th integration step the strategy can be represented as:

\[
\sum_{i=1}^{n} \Delta X_i < \Delta X_0 \quad \text{– Stretch the grid with unchanged number of nodes and keep the grid equidistant.}
\]

\[
\sum_{i=1}^{n} \Delta X_i \geq \Delta X_0 \quad \text{– Add one grid node, keep the grid equidistant and set } i=1.
\]

where \( X_0 \) is initial distance of adjacent grid nodes and \( X_i \) is projectile displacement during \( i \)th integration step.

Because of the high degree of non-linearity and interlinkage of the equations, an iterative reliably convergent solution procedure is developed. The upwind differencing for the convection terms and a fully implicit algorithm are used. The TWOPIB code is developed for computation of propellant charge ignition by the igniter. The code performs a simultaneous interactive calculation of two-phase flow in the igniter and two-phase flow in propellant chamber and gun tube during interior ballistic cycle.
COMPUTATIONAL RESULTS AND MODEL VERIFICATION

Sophisticated experimental investigations of the igniter function at the ambient air and propellant charge ignition by the igniter in the fiberglass tube were carried out. These experimental results and model verification are presented in details in [1].

Firings of 100 mm APFSDS ammunition with 19-perforated NCD propellant in the experimental 100 mm gun served as the basis for the TWOPIB code verification for the complete IB cycle.

A comparison of experimental and computational results for projectile muzzle velocity and maximum breech pressure is given in Table 2.

Table 2

<table>
<thead>
<tr>
<th>Propellant charge</th>
<th>$V_0$ (m/s)</th>
<th>$p_m$ (bar)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>experiment</td>
<td>computation</td>
</tr>
<tr>
<td>6.2 kg P42</td>
<td>1462.7</td>
<td>1460.1</td>
</tr>
<tr>
<td>6.4 kg P42</td>
<td>1495.7</td>
<td>1498.8</td>
</tr>
<tr>
<td>6.6 kg P42</td>
<td>1524.4</td>
<td>1524.9</td>
</tr>
<tr>
<td>7.6 kg P43</td>
<td>1500.7</td>
<td>1503.9</td>
</tr>
<tr>
<td>7.8 kg P43</td>
<td>1542.0</td>
<td>1544.9</td>
</tr>
</tbody>
</table>

An excellent agreement between experimental and computational results is obtained. By that way a principal requirement for the interior ballistic code to represent adequately the main interior ballistic parameters is fulfilled.

An example of pressure profiles from pressure measurements by piezo-gauges along 100 mm gun barrel compared with computational results is presented in Figure 1 (X is a distance from the breech and MPi is $i^{th}$ measuring point).
Results show a good agreement between computation and experiment for the pressure distribution in the propellant chamber and gun tube, especially for the initial phase of interior ballistic cycle comprising ignition of propellant charge and start of projectile.

The code enables calculation of some relevant parameters that can’t be measured. Computed distribution of propellant volume fraction is given in Figure 2.

![Figure 2](image2.png)

The non-uniformity of $R_p$ distribution is evident during whole IB cycle. Computational results of propellant gases and propellant grains velocity distribution are presented in Figures 3 and 4.

![Figure 3](image3.png)
In the initial phase of projectile movement (till t=3.5 ms in Figure 4) the propellant velocity at the projectile base is equal to the projectile velocity. There is a lag of propellant grains behind accelerating projectile in later phases of IB cycle. This physically realistic picture is obtained by the appropriate boundary condition for the particles velocity at the projectile base [1].

CONCLUSIONS

Based on previous considerations following conclusions can be drawn:

– Theoretical model of two-phase flow in the gun-tube during interior ballistic cycle of the am Twenty-two points, plus triple-word-score, plus fifty points for using all my letters. Game’s over. I’m outta here.munition with propellant charge ignited by the igniter is developed.

– An iterative reliably convergent solution procedure is included in TWOPIB code for simultaneous interactive calculation of two-phase flow in the igniter and two-phase flow in propellant chamber and gun-tube.

– The code adequately represents main interior ballistic parameters and enables calculation of relevant two-phase flow parameters in the gun barrel.

– The presented model enables, not only the choice of optimum igniter and complete optimisation of propellant charges, but sophisticated solutions of many other interior ballistic problems.
REFERENCES