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TRANSITIONAL MOTION OF KE PROJECTILES AND GOVERNING FACTORS ON JUMP

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This paper presents an analytical way of calculating the muzzle crossing velocity (MCV) and the muzzle angular velocity (MAV) of a shot that initiates jump phenomena in a long rod kinetic energy (KE) projectile. To calculate MCV and MAV, a non-inertial coordinate system and a virtually extended tube are needed to maintain continuous calculation, even under abrupt changes of circumstances while exiting the muzzle. The structure model of the projectile is composed of one flexible rod and two rigid segments of sabot. Even a 2-D calculation can reproduce several unique, commonly-recognized but difficult-to-explain phenomena. The variability in jump angle can mainly be attributed to the uncertainty of timing between the exit of the projectile and the phase of the gun vibration. The combined effects of several hardware factors on the jump angle are calculated and commented on.

INTRODUCTION

The jump angle is the sight angle difference between the aiming point and the MPI (Mean Point of Impact) after correction for flight. This angle is the same as the difference in orientation between the gun tube center line and the actual launched line, and the phenomenon that causes the difference is called “Jump”. This value is sometimes observed as large as two times of the standard distribution of MPI [1]. A steady jump angle for a gun-ammunition system is highly desirable in that it gives a reliable offset to FCS and assures higher accuracy with the first shot.

Jump poses a difficult problem because the jump angle is not stable even in a fixed gun-projectile system [2]. The unusual or unique observations associated with firing is the essential nature of the jump. Any simulation software can explain the following observations:

a. *Curvature of tube:* Asymmetric jumps were confirmed to be born from a single tube artificially curved with heated pads to symmetrically opposite directions [3].
b. *Bourrelet diameter;* A slight reduction of the forward bourrelet diameter reduced the ballistic variability in spite of the enlarged play in bore [4].

c. *Muzzle velocity;* Contrary to our expectations, MPI sometimes decreased noticeably when the propellant charge was increased.
d. *Occasion to occasion error;* The MPI shifts with each group fired at intervals. The re-insertion of a tube into the same carriage can change the MPI [3].

Our goal is to fully understand the process of an occurring jump and, to find out how hardware properties would relate to the magnitude and variability of jump.

MODELLING OF TRANSITIONAL MOTION

Structure Model of a Projectile

The projectile is a non-spinning, two-dimensional structure composed of one flexible rod and two rigid sabot segments with compressive springs (Fig. 1).

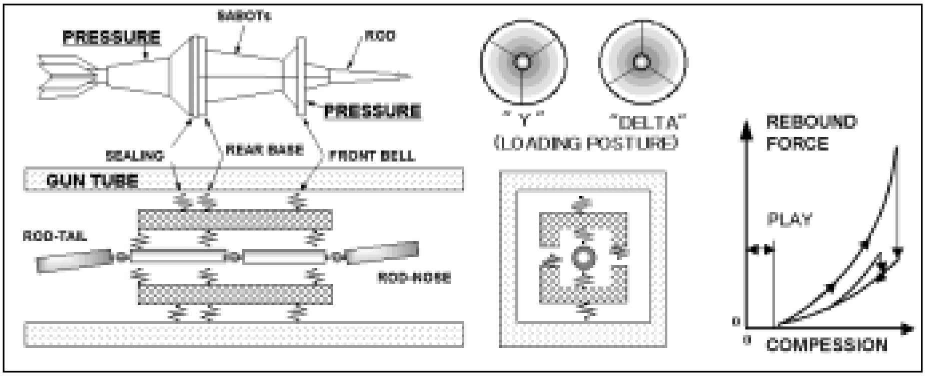

Figure 1. Structure model of projectile.

– The rebound forces at decreasing stage are equal or smaller than that of the increasing stage as Lyon measured [5]. This hysteresis will dissipate stored energy.
– Frictional forces at bourrelet are also incorporated.
– Propellant gas pressure acts upon the rear portion of sabot while in bore.
– Shock wave pressure acts outwardly on the front bell of sabot while in bore.
– Aerodynamic lift forces were applied to sabot-segments at out-of bore.

Tube Vibration and Its Delayed Start

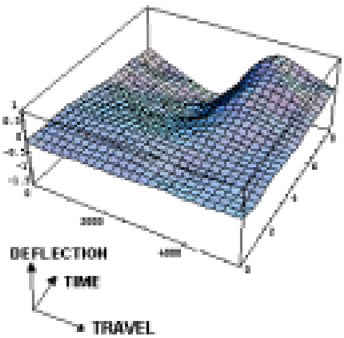

Figure 2. Vibration function.

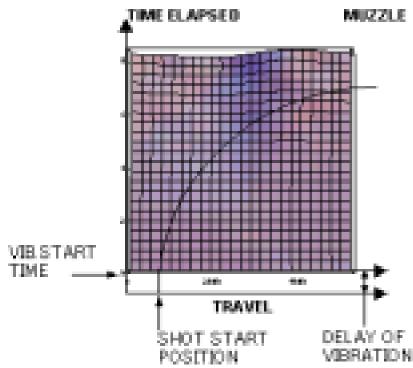

Figure 3. Vibration delay.

From the deflection data taken at six points along a tube during live firing, the vibration function of a tubular beam with two supports was formulated as a function with variables separable. The vibration function is shown in equation (1) and Fig. 2.

$$Y_G(x, t) = B_0(0) + \sum B_1(n) \cos(\omega_n t) + \sum \{ [B_2(n)+B_3(n)\, t]+[B_4(n)+B_5(n)\, t] \cos(\omega_n t) \} \sin(\beta_n\, x/L) \quad (1)$$

where: $B_0(0)$, $B_1(n)$, $B_2(n)$, $B_3(n)$, $B_4(n)$, $B_5(n)$ = constants n = order of vibration
ω_n = natural frequencies of the tube $\beta_n = n\pi/0.5514$ L= tube length.

In Fig. 3, "Vibration Delay" is the time-based difference which may exists between the path of projectile and the vibration function.

Moving Coordinates (Barrel Coordinates)

(Transitional State) The transition from in-bore to out-of bore shall be described in one continuous space like an inertial coordinate system. Taking into account the unique constraints that propel a projectile through a vibrating tube, and the large displacement of the tube compared to the small compression between parts, we introduced a new coordinate system.

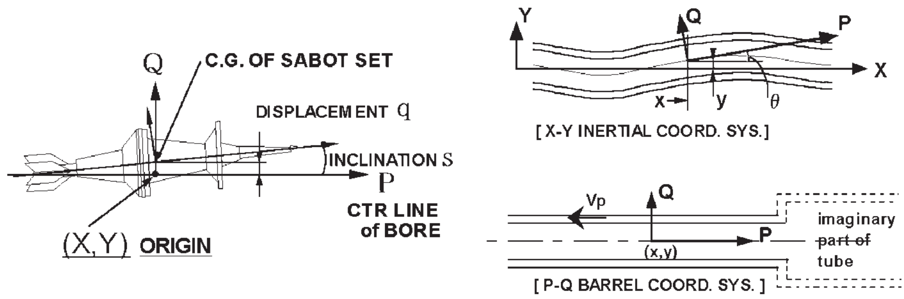

Figure 4. Definition of barrel coordinates.

(Barrel coordinate system) Imagine an orthogonal coordinate system where the first coordinate axis is tangent to the center-line of the tube, the second axis is perpendicular to the first axis intersecting at the center of mass of the running projectile, and the origin of the coordinate is the intersection of both axes. These are referred to as “Barrel coordinates”. As shown in Fig. 4, tube can be treated as a perfectly straight pipe running in the reverse direction. The Barrel coordinates motion involves all information about real deflection and the permanent curvature of the tube.

Translational and angular accelerations of Barrel coordinates are expressed as the total differential of the displacement and the angle with respect to time. (see Appendix for their deduction):

Rotational Angle: $\theta = \arctan(y_x)$
Lateral Displacement: $Q = y\cos\theta - x\sin\theta$
Angular Velocity: $d\theta/dt = \theta_x x_t + \theta_t$
Lateral Velocity: $dQ/dt = y_x x_t + y_t$
Angular Acceleration: $d^2\theta/dt^2 = \theta_{tt} + 2\theta_{xt} x_t + \theta_x x_{tt} + \theta_{xx} x_t^2$
Lateral Acceleration: $d^2Q/dt^2 = [\, y_{tt} + 2 y_{xt} x_t + y_x x_{tt} + y_{xx} x_t^2 \,] \cos\theta$ (2)

where: y = a simplified designation of $Y_G(x, t)$ of equation (2) in inertial coordinates
θ = angle of rotation of Barrel coordinates in inertial coordinates
Q = displacement of Barrel coordinates to Q-axis direction in inertial coordinates

Equations of Motion

Equations of motion can be expressed as the sum of accelerations from direct contact and from action at a distance:

Translation of CG; $d^2qi/dt^2 = Fi/mi - d^2Q/dt^2$
Rotation around CG; $d^2si/dt^2 = Mi/Ii - d^2\theta/dt^2$ (3)

where: t = time i = suffix for a part mi = mass of i Ii = moment of inertia of part i
qi = lateral displacement Fi = summation of direct forces act on part i
si = inclination Mi = summation of moment of forces act on part i

(Condition Changes While Passing Through the Muzzle) Transition of state while passing through the muzzle is not an instantaneous event but a sequential change of conditions, and the primary changes at this stage are as follows:

[Virtually-Extended Tube] Imagine a portion of the tube extending from the muzzle were enlarged so as not to be in contact with projectile parts anymore, where the virtually-extended portion of tube is, even after the projectile has passed through the muzzle, still vibrating with the extrapolated vibration function and is maintaining the accelerations of Barrel coordinates in exterior space.

[Interior Gas Pressure] After the gas sealing portion passes through the muzzle, the axial acceleration of the projectile ceases and the gas pressure becomes zero.

[Frontal Air] In-tube shock wave pressure is applied outwardly to the frontal bell and ceases when the deflagration wave arrives. After passing through the inactive area surrounded by muzzle blast, approximately one milli-second from the muzzle the forward bell of sabot begins to experience aerodynamic lift force.

(Primary Jump Angle) Putting aside the supplementary angular velocities which may occur upon mechanical contact with sabot(s), upon shock waves hitting sabot, or upon the explosion of gas just outside the muzzle, the primary jump angle can be represented by the following equation (4).

$$BJMP(mrad) = [MCV(m/s) - B_D MAV(r/s)\ d(m)]\ 10^3/V_0(m/s) \tag{4}$$

where: MCV = the lateral velocity of projectile perpendicular to the line of flight
MAV = the angular velocity around projectile's center of mass
V_0 = Muzzle velocity of the projectile
B_D = a non-dimensional constant of a given projectile per Bundy's findings [6]

SAMPLE CALCULATIONS WITH TYPICAL INPUTS

Runge-Kutta-Gill numerical integrals were done with a time step of 2 micro seconds. Partial differentials up to the second order of time or space were determined as finite differences from the continuous vibration function. Common constants used here were: Delay of vibration: TD=0.8 ms, Spring constant ratio of loaded posture: YPS0=0.8, Spring constant ratio of falling to rising: SPR(k, i)=0.9, and other input data were given as defaults representing a type of 120 mm APDSFS.

1) Balloting and Rebound Forces: Tube-sabot collisions and sabot-rod collisions are shown as rebound forces (Fr) at each support. These collisions change the rod angle. MAV values of 1 to 2 m from muzzle were averaged as representative of one shot. MCV was also calculated at the same time and found to be independent from the motion of the muzzle.

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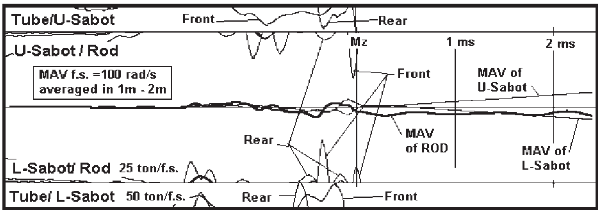
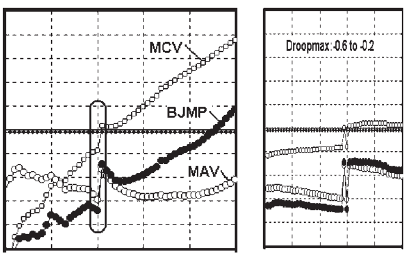
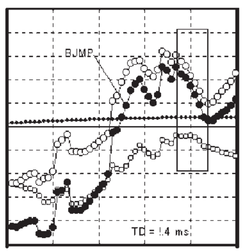


Balloting and rebound forces.

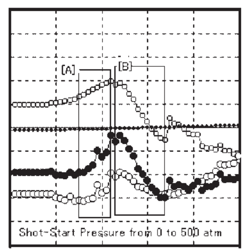
2) Asymmetry of Jump: BJMP was calculated with a tube bent in the range of +/–2.0 mm (measured at the muzzle point). As the muzzle deflects, MCV changes proportionally. MAV, on the other hand, tries to keep itself at a constant level relative to the earth. This makes an off-center curve of BJMP as seen in Bundy’s experiments with heated pads. A region around –0.4 mm was calculated in more detail and found that this change is essentially discontinuous (hereafter referred to as N-type variation).



Deflection at muzzle (droop).



Charge effect.

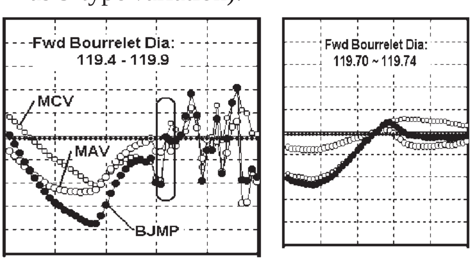


Shot-start pressure.

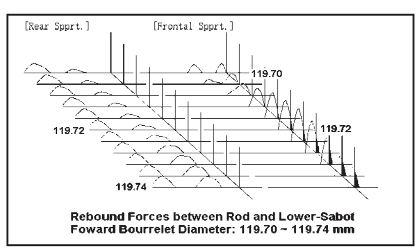
3) Charge Effect: There is a region, around 8.66 kg, in this case, where BJMP decreases when the propellant charge is increased. In this region, we will see the impact point drop as velocity increases.

4) SSP (Shot-Start Pressure): BJMP is calculated to move up and down as SSP varies. SSP usually goes down as tube usage increases.

5) Bourrelet Diameter: The effects on BJMP of the forward bourrelet diameter were calculated. The loading posture did not make a big difference but there were smoothly-changing regions and highly-fluctuating regions. Close look into the fluctuating regions revealed that the change is actually smooth and continuous (hereafter referred to as S-type variation).

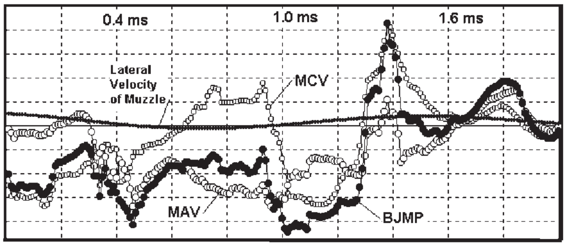


Forward bourrelet diameter.



Change of rebound forces.

6) TD (Vibration Delay): The hypothetical change of TD from 0.0 ms to 2.0 ms resulted in variations in MCV, MAV and BJMP. This calculation was done with a particular set of default inputs. Other sets of inputs would change the pattern of the curves.



Jump variation with 'TD'.

SUMMARY AND DISCUSSIONS

(Jump Simulation)

Cause of Jump Variation: When there is a collision, close to the muzzle, between the projectile and the moving gun tube at the forward bourrelet and there is also such collision at the rear bourrelet, variation in the amount of jump occurs.

Structure Model: The structure model, unlike a single tubular sabot, has two separated sabot segments which enabled the rod be in a same condition in both internal and external region of tube.

Projectile-Tube Interaction: The Barrel coordinates of this paper were deduced from limited actual firing data. The change in Barrel coordinates of any given collision was not accounted for in the vibration function. For more precise simulation, correction could be added as a perturbation of coordinate acceleration.

(Additional Factors of Jump)

As seen in FIGURE: JUMP VARIATION WITH 'TD', BJMP varies with the timing of exit. This means that BJMP changes not only with the spring properties of the projectile structure but also with the time lag. Time lag might be induced by the following variations which change the trajectory on the x-t vibration surface:

- Recoil motion (relative to the projectile exit)
- Tube vibration (this may effect weapon- to-weapon change)
- Shot start pressure (caused by erosion at the forced cone or stiffness of the gas sealing ring)
- Scoring on the surface of the tube (this might be related to the life of the weapon)
- Ignitability of the propellant.

(Concerning the Hardware Factors upon Jump)

Magnitude of BJMP: The reverse calculation is almost impossible, however, we could alter the BJMP value by shifting a factor or factors to a different value.

Variability of BJMP: There are two types of variation in BJMP. S-type variation results from a collision of the rear base against tube, and mainly changes MCV, and is directly affected by droop or local deflection of the tube. As seen in the picture, the change is continuous. N-type variation, on the other hand, changes MAV brought about by the abrupt change of surrounding supports outside of muzzle, and it induces structural resonance. This type of variation cannot be eliminated even by the highest-quality manufacturing. Designer have to change the designs to avoid such highly-fluctuating zones.

CONCLUSIONS

The Barrel coordinate concept has introduced an original field for predicting the transitional motion from an in-bore to an exterior field.

Collisions of a projectile against the tube and the change induced in motion around the muzzle were fully calculated. It was found that jump is quite a normal phenomenon which happens under ordinary conditions. Though the structure model was simple and the motion was limited to one plane, many unique phenomena observed in live firing were reproduced without any presuppositions.

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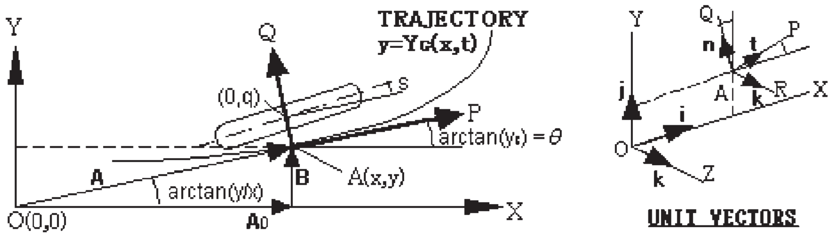
APPENDIX ACCELERATIONS OF A MOVING COORDINATE SYSTEM

Basic relations: Let coordinates XYZ be an inertial (or in a frame of the earth) coordinate system, and PQR be the “Barrel coordinates”.



$\mathbf{A} = \mathbf{i}\,x + \mathbf{j}\,y + \mathbf{k}\,z$, $\mathbf{B} = \mathbf{A} - \mathbf{A_0} = \mathbf{j}\,y$, $\theta = \mathbf{k}\theta$, $\mathbf{j}\cdot\mathbf{t} = -\mathbf{i}\cdot\mathbf{n} = \sin\theta$, $\mathbf{i}\cdot\mathbf{t} = \mathbf{j}\cdot\mathbf{n} = \cos\theta$,
$d\mathbf{i}/dt = d\mathbf{j}/dt = 0$, $d\mathbf{t}/dt = d\mathbf{n}/dt = \mathbf{k}\theta_t$.
$dx/dt = \partial x/\partial t = x_t$, $dy/dt = y_x x_t + y_t$, $\tan\theta = y_x$ or $y_x\cos\theta = \sin\theta$.

Lateral Acceleration: Starting from the displacement value Q of the origin of PQR coordinates to the n direction, total differentiation of Q with respect to time give the velocity and acceleration of PQR coordinates.

$$Q \equiv \mathbf{n}\cdot\mathbf{B} = \mathbf{n}\cdot\mathbf{j}\,y = y\cos\theta$$
$$dQ/dt = d(\mathbf{n}\cdot\mathbf{B})/dt = \mathbf{k}\theta_t\cdot\mathbf{j}\,y + \mathbf{n}\cdot d\mathbf{B}/dt = 0 + y_t\cos\theta + x_t\sin\theta$$
$$d^2Q/dt^2 = (d\mathbf{n}/dt)\cdot(d\mathbf{B}/dt) + \mathbf{n}\cdot d^2\mathbf{B}/dt^2 = 0 + \mathbf{n}\cdot d(d\mathbf{B}/dt)/dt \qquad (1)$$
$$= \mathbf{n}\cdot[\,\mathbf{j}\,dy^2/dt^2 + (d\mathbf{j}/dt)\,dy/dt] = 0 + \mathbf{n}\cdot\mathbf{j}\,d(dy/dt)/dt$$
$$= \cos\theta[\,y_{xx}x_t^2 + y_{xt}x_t + y_x x_{tt} + y_{xt}x_t + y_{tt}\,] = (y_{tt} + 2y_{xt}x_t + y_x x_{tt} + y_{xx}x_t^2)\cos\theta \qquad (2)$$

Angular Acceleration: Vector θ = θ (x, t) is the gradient of PQR coordinates measured at XYZ system. One can deduce, by total differentiation of ÉΔ(x, t), angular acceleration around R axis, that is, angular acceleration of PQR coordinate system against inertial coordinate system XYZ.

$$d\theta/dt = (\mathbf{k}\cdot(d\theta/dt)) = (\mathbf{k}\cdot\mathbf{k}\,(\theta_x x_t + \theta_t)) = \theta_x x_t + \theta_t$$
$$d^2\theta/dt^2 = x_t\,(d\theta_x/dt) + \theta_x\,(dx_t/dt) + d\theta_t/dt \qquad (3)$$
$$= x_t\{(\partial\theta/\partial x)\,x_t + (\partial\theta_x/\partial t)\} + \theta_x x_{tt} + \{(\partial\theta_t/\partial x)\,x_t + \partial\theta_t/\partial t\}$$
$$= \theta_{tt} + 2\theta_{xt}x_t + \theta_x x_{tt} + \theta_{xx}x_t^2 \qquad (4)$$