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ADIABATIC DEPRESSURISATION OF VENTED VESSELS

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Vented vessels are used to determine propellant performance in a safe and operationally correct environment compared to closed vessels. Propellant is burnt within a chamber and the gases vented to atmosphere through a narrow throat. Vented vessels can simulate the operation of energetic devices such as ejection cartridges, rocket motors or base bleed systems. Computer modelling of steady-state combustion is simple and for little loss in accuracy the system can be regarded as burning at constant flame temperature. However the flame temperature is generally assumed to remain constant during the depressurisation phase from full combustion to atmospheric pressure. This paper shows that this assumption introduces errors and gives a lumped parameter theoretical model for Adiabatic Depressurisation. This predicts temperature and pressure with time for the depressurisation of any vented vessel. The model is shown to be in good agreement with trials.

INTRODUCTION

In all vented vessel propellant assessment methods the author has encountered, the standard assumptions are a constant flame temperature (isothermal) throughout the burn including depressurisation, and a negligible covolume. These assumptions produce significant errors in determining both the propellant's force constant and the overall system performance. The level of error depends on the burn to depressurisation times.

In this paper the depressurisation of a vented vessel is predicted based on adiabatic and then isothermal conditions. Both are derived initially for a true covolume and then subsequently simplified assuming a negligible covolume. The different models are then assessed.

THEORETICAL DERIVATION OF THE MODELS

(All symbols are defined at the end of the paper)

General Gas Equations

Starting with the Equation of State:

$$P(V - m\,\eta) = n\,R\,T \tag{1}$$

But (adjusting for SI units):

$$n = (1000\,m) / M \tag{2}$$

Let:

$$R_S = n\,R / m = 1000\,R / M \tag{3}$$

then

$$P\,(V - m\,\eta) = m\,R_S\,T \tag{4}$$

Also let:

$$\lambda = R_S\,T \tag{5}$$

hence (4) becomes:

$$P\,(V - m\,\eta) = m\,\lambda \tag{6}$$

Under initial conditions (5) becomes:

$$\lambda_O = R_S\,T_O \tag{7}$$

Throughout this paper the ratio of specific heats (γ) is assumed to be constant. From Reference [1] we define for supersonic choked flow, the Characteristic velocity (c) as:

$$c = -A_t\,P / (dm/dt) = (R_S\,T)^{1/2} / (\gamma^{1/2}\,(2/(\gamma+1))^{(\gamma+1)/2(\gamma+1)}) \tag{8}$$

where A_t is the effective gas throat exit area. Rearranging (5) and (8) gives:

$$c = (\,((\gamma+1)/2)^{(\gamma+1)/(\gamma+1)}(\lambda/\gamma)\,)^{1/2} \tag{9}$$

Hence:

$$\lambda = \gamma\,c^2 \cdot ((2/(\gamma+1))^{(\gamma+1)/(\gamma-1)} \tag{10}$$

For unchoked flow we have from Reference [1], where P_a is the atmospheric pressure:

$$(dm/dt)\,\lambda^{1/2} / (A_t\,P) = -(2\gamma/(\gamma-1))^{1/2}\,(P_a/P)^{1/\gamma}\,(1 - (P_a/P)^{(\gamma-1)/\gamma})^{1/2} \tag{11}$$

Rearranging and by defining function f~P), References [1, 2] give

$$(dm/dt) / (A_t\,P) = -\lambda^{-1/2}\,(2\gamma/(\gamma-1))^{1/2}\,(\,(P_a/P)^{2/\gamma} - (P_a/P)^{(\gamma+1)/\gamma})^{1/2}$$

$$= -f(P) / c \tag{12}$$

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Substituting for c from (9) into (12) gives for subsonic flow:

$$f(P) = \left(\left((\gamma+1)/2\right)^{(\gamma+1)/(\gamma-1)} \cdot (2/(\gamma-1)) \cdot \left((P_a/P)^{2/\gamma} (P_a/P)^{(\gamma+1)/\gamma} \right) \right)^{1/2} \tag{13}$$

and

$$(dm/dt) = -A_t\, P\, f(P) / c \tag{14}$$

For supersonic flow (14) becomes:

$$(dm/dt) = -A_t\, P / c \tag{15}$$

Equation (14) becomes (15) when f(P) = 1. Thus (14) can be used for general flows by setting an appropriate value for the function f(P), as described in the next section.

Flow Correction Function, f(P)

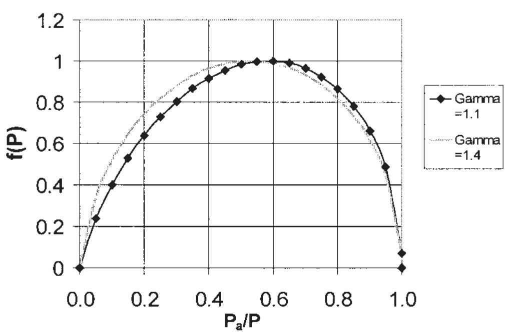

Figure 1. Function f(P) vs pressure ratio.

Function f(P) in (13) can be thought of as a correction factor for unchoked restricted flow. This function is plotted for various values of γ in Figure (1). Values of f(P) fall into three bands (a, b, c) depending on the flow regime, itself dependent on the ratio (P_a/P).

(a) When (P_a/P) is large, approximately 0.6 to 1.0, flow is subsonic and follows equations (13, 14).

(b) When flow is sonic, (P_a/P) has a critical value and f(P) = 1, following equations (13, 14). Critical flow occurs when function f(P) is at a maximum.

Differentiating (13) gives:

$$df(P)/d(P_a/P) = ((\gamma+1)/2)^{(\gamma+1)/2(\gamma-1)} (2/(\gamma-1))^{1/2} \left((2/\gamma)(P_a/P)^{(2-\gamma)/\gamma} - ((\gamma+1)/\gamma)(P_a/P)^{1/\gamma} \right) / \left(2 \cdot \left((P_a/P)^{2/\gamma} - (P_a/P)^{(\gamma+1)/\gamma}\right)^{1/2} \right) \tag{16}$$

If P_a is a constant, setting $df(P)/d(P_a/P)$ to zero gives f(P) =1 and a critical value for P defined as P_c (17), where:

$$P_a/P_c = (2/(\gamma+1))^{\gamma/(\gamma-1)} \tag{17}$$

(c) When (P_a/P) is small (0.0–0.5), flow is supersonic, the throat becomes choked, with f(P) equal to 1, regardless of the smaller value of the function (see Figure 1). Flow is therefore represented by equation (15).

Hence for a general solution (Reference [2]), equation (13) becomes:

if P < Pc *Subsonic flow*

$$f(p) = (((\gamma+1)/2)^{(\gamma+1)/(\gamma-1)} \cdot (2/(\gamma-1)) \cdot ((P_a/P)^{2/\gamma} - (P_a/P)^{(\gamma+1)/\gamma}))^{1/2} \quad (13a)$$

else if $P = P_c$ *Sonic flow*

$$f(P) = 1 = (((\gamma+1)/2)^{(\gamma+1)/(\gamma-1)} \cdot (2/(\gamma-1)) \cdot ((P_a/P)^{2/\gamma} - (P_a/P)^{(\gamma+1)/\gamma}))^{1/2} \quad (13b)$$

else $P > P_c$ *Supersonic flow*

$$f(p) = 1 \quad (13c)$$

Generally the vessel will be at a much greater pressure than atmospheric, so flow is supersonic, following equation (13c). This can be assumed in subsequent equations.

Depressurisation of a Vented Vessel

Differentiating (4) with respect to time(t) and assuming V, η, R_S, M and γ are constants, gives:

$$V (dP/dt) = m R_S (dT/dt) + R_S T (dm/dt) + (dP/dt) m \eta + P \eta (dm/dt) \quad (18)$$

Substituting from (14) into (18) gives the *General Depressurisation Equation (19):*

$$V (dP/dt) = m R_S (dT/dt) - R_S T A_t P f(P) / c + (dP/dt) m \eta - P^2 \eta A_t f(P) / c \quad (19)$$

Assuming Adiabatic Conditions

From Reference [1] we have for Adiabatic conditions:

$$T = T_O P_O^{(1-\gamma)/\gamma} P^{(\gamma-1)\gamma} \quad (20)$$

Subscripts o refer to the initial conditions (i.e. pressure and temperature of the gas at the commencement of vented vessel depressurisation). Differentiating (20) with respect to time (t) gives:

$$(dT/dt) = T_O P_O^{(1-\gamma)/\gamma} ((\gamma-1)/\gamma) P^{-1/\gamma} (dP/dt) = k_1 P^{-1/\gamma} (dP/dt) \quad (21)$$

where k_1 is defined as:

$$k_1 = ((\gamma-1)/\gamma) T_O P_O^{(1-\gamma)/\gamma} \quad (22)$$

Let:

$$k_2 = T_O P_O^{(1-\gamma)/\gamma} \quad (23)$$

then from (20) we have:

$$T = k_2 P^{(1-\gamma)/\gamma} \quad (24)$$

Substituting from (21, 24) for (dT/dt) and T into (19) gives:

$$V\,(dP/dt) = m\,R_S\,k_1\,P^{-1/\gamma}\,(dP/dt) - R_S\,A_t\,f(P)\,k_2\,P^{(2\gamma-1)/\gamma} / c + m\,\eta\,(dP/dt) - P^2\,\eta\,A_t\,f(P)/c \qquad (25)$$

But from (5, 9):

$$1 / c = \left((2/(\gamma+1))^{(\gamma+1)/(\gamma-1)} \cdot (\gamma/(R_S\,T)) \right)^{1/2} \qquad (26)$$

Let:

$$k_3 = \left((2/(\gamma+1))^{(\gamma+1)/(\gamma-1)} \cdot (\gamma / R_S) \right)^{1/2} \qquad (27)$$

then

$$c = T^{1/2} / k_3 \qquad (28)$$

Under initial conditions k_3 is given by:

$$k_3 = T_O^{1/2} / c_O = T^{1/2} / c \qquad (29)$$

Substituting for T in (20, 28) gives:

$$1 / c = k_3\,T_O^{-1/2}\,P_O^{(\gamma-1)/(2\gamma)}\,P^{(1-\gamma)/(2\gamma)} \qquad (30)$$

or

$$1 / c = k_4\,P^{(1-\gamma)/(2\gamma)} \qquad (31)$$

where

$$k_4 = k_3\,T_O^{-1/2}\,P_O^{(\gamma-1)/(2\gamma)} = P_O^{(\gamma-1)/(2\gamma)} / c_O \qquad (32)$$

Substituting for c from (31) in (25) gives:

$$V(dP/dt) = m\,R_S\,k_1\,P^{-1/\gamma}\,(dP/dt) - R_S\,k_2\,P^{(2\gamma-1)/\gamma}\,A_t\,f(P)\,k_4\,P^{(1-\gamma)/(2\gamma)} + m\,\eta\,(dP/dt) - P^2\,\eta\,A_t\,f(P)\,k_4\,P^{(1-\gamma)/(2\gamma)} \qquad (33)$$

Defining

$$k_5 = R_S\,k_2\,k_4\,A_t \qquad (34)$$

and

$$k_6 = \eta\,A_t\,k_4 \qquad (35)$$

then:

$$V\,(dP/dt) = m\,R_S\,k_1\,P^{-1/\gamma}\,(dP/dt) - k_5\,f(P)\,P^{(3\gamma-1)/(2\gamma)} + m\,\eta\,(dP/dt) - k_6\,f(P)\,P^{(3\gamma+1)(2\gamma)} \qquad (36)$$

Rearranging equation (4) gives:

$$m = P\,V / (R_S\,T + P\,\eta) \qquad (37)$$

Substituting (24) into (37) gives:

$$m = V/(k_2\,R_S\,P^{-1/\gamma} + \eta) \qquad (38)$$

Substituting for m from (38) into (36) gives:

$$V\,(dP/dt) = (R_S\,k_1\,V\,P^{-1/\gamma}) / (k_2\,R_S\,P^{-1/\gamma} + \eta) \cdot (dP/dt) - k_5\,P^{(3\gamma-1)/(2\gamma)}\,f(P) + \eta\,(dP/dt)\,V / (k_2\,R_S\,P^{-1/\gamma} + \eta) - k_6\,f(P)\,P^{(3\gamma+1)\,(2\gamma)} \quad (39)$$

This gives equation (40) the *Adiabatic Depressurisation Model*

$$(dP/dt) = f(P) \cdot (-k_5\,P^{(3\gamma-1)/(2\gamma)}\,k_6\,P^{(3\gamma+1)/(2\gamma)}) / (V - (R_S\,k_1\,V) / (k_2\,R_S + \eta\,P^{1/\gamma}) - (V\,\eta) / (k_2\,R_S\,P^{-1/\gamma} + \eta)) \quad (40)$$

In general the vessel will be at a much higher pressure than atmospheric, so f(P) = 1. Equation (40) can be greatly simplified by setting the covolume to zero ($\eta = 0$, $k_6 = 0$):

$$(dP/dt) = f(P) \cdot (-k_5\,P^{(3\gamma-1)/(2\gamma)}) / (V - (k_1\,V) / (k_2)) \quad (41)$$

Becoming:

$$(dP/dt) = f(P) \cdot (-\gamma\,k_5 / V) \cdot P^{(3\gamma-1)/(2\gamma)} \quad (42)$$

Assuming Isothermal Conditions

From (19) if we assume a constant Flame Temperature for Isothermal Depressurisation then $(dT/dt) = 0$ and $T = T_O$ (the initial starting condition). From (28), c becomes c_O as in (29). Equation (19) then becomes:

$$V\,(dP/dt) = -R_S\,T\,A_t\,P\,f(P) / c + (dP/dt)\,m\,\eta - P^2\,\eta\,A_t\,f(P) / c \quad (43)$$

hence:

$$(dP/dt)\,(V - m\,\eta) = -R_S\,T_O\,A_t\,P\,f(P) / c_O - P^2\,\eta\,A_t\,f(P) / c_O \quad (44)$$

Substituting from (7):

$$(dP/dt) = (P\,A_t\,f(P) / c_O) \cdot (-\lambda_O - P\,\eta) / (V - m\,\eta) \quad (45)$$

From (6, 7) we have, at constant T:

$$P(V - m\,\eta) = m\,\lambda_O \quad (46)$$

Rearranging gives:

$$m = (PV) / (\lambda_O + P\,\eta) \quad (47)$$

From (45) substituting for m (47) gives (48), the *Isothermal Depressurisation Model*

$$(dP/dt) = (P\,A_t\,f(P) / c_O) \cdot (-\lambda_O - P\,\eta) / (V - (P\,V\,\eta) / (\lambda_O + P\,\eta)) \quad (48)$$

If the covolume is set to zero ($\eta = 0$), equation (48) becomes simply:

$$(dP/dt) = (P\,A_t\,f(P) / c_O) \cdot (-\lambda_O / V) \quad (49)$$

DISCUSSION

Figure 2 shows depressurisation predicted by the adiabatic model (with a true covolume) and the standard isothermal model (with negligible covolume) together with a trial firing of an ejection cartridge containing 13 g of double based propellant. It shows that a significant error in depressurisation rate and pressure integral is generated by the isothermal assumptions. This pressure integral is often used in determining the propellant's force constant. Table 1 emphasises the difference between the models.

Table 1. Comparison of Models

Vented Vessel Comparison shown graphically in Figure 2	Pressure Integral (MPa-ms)	Ratio of Predicted to Actual Force Constant
Adiabatic model with covolume	4285.23	0.95
Adiabatic model with zero covolume	5643.10	1.49
Isothermal model with covolume	4923.12	1.19
Isothermal with zero covolume	6376.91	1.84

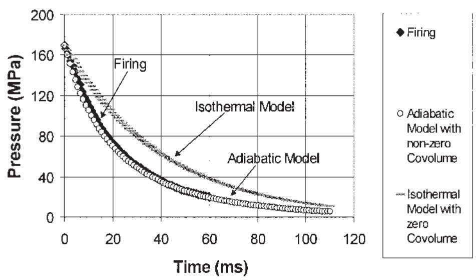

Figure 2. Depressurisation with Time.

To avoid errors it is recommended that pressure integrals are used for system performance modelling rather than derived force constants, unless the standard isothermal models used have been replaced by adiabatic ones.

NOMENCLATURE

- A_t Effective gas throat exit area (m^2)
- c Characteristic velocity (m/s)
- c_O Initial characteristic velocity (m/s)
- η Gas covolume (m^3/kg)
- f(P) Correction factor for unchoked gas flow; for choked flow f(P)=1
- γ Ratio of specific heats
- k_n System constants, $k_1 - k_6$ as defined above
- λ Force constant (J/kg)
- λ_O Initial force constant (J/kg)
- m Mass of gas (kg)
- M Gas molecular weight (g/mol)
- n Moles of gas (mol)
- P Gas pressure at any instant during depressurisation (Pa)
- P_a Atmospheric pressure (Pa)
- P_c Critical gas pressure at which flow is sonic (Pa)
- P_O Initial gas pressure (Pa)
- R Universal gas constant (J/mol/K)
- R_S Modified gas constant (J/kg/K)
- t Time(s)
- T Gas temperature at any instant during depressurisation (K)
- T_O Initial gas temperature (K)
- V Volume of vessel (m^3)

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REFERENCES

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2. W F. Hughes & J.A. Brighton, "Fluid Dynamics", *McGraw-Hill*