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REFERENCE CORRELATIONS FOR TUNGSTEN LONG RODS STRIKING SEMI-INFINITE STEEL TARGETS

R. Subramanian and S. J. Bless

The Institute for Advanced Technology, 3925 W. Braker Ln., Austin, Texas 78759, USA

Several analytical forms are proposed for empirical correlations of normalized penetration depth (P/L) in semi-infinite targets. The functional forms are based on the widely accepted Lanz-Odermatt equation. The modifications alter the transition to the hypervelocity asymptote and add a scale-dependence term. The equations are fit to a compilation of tungsten long-rod data Implications of the resulting parameters are discussed.

INTRODUCTION

Empirical correlations for normalized penetration depth (P/L) as a function of impact velocity (v) are essential in systems studies and the evaluation of complex armors or novel penetrators. For eroding penetrators, one functional form that has enjoyed recent popularity is a two parameter version of the Lanz-Odermatt function [1]. This function provides a reasonable fit to the observed trends over a wide velocity range and the parameters can be related to material properties.

In this paper we propose several modified forms of the Lanz-Odermatt function. Two forms add one parameter, one which alters the transition from lower to higher velocity behavior and one which adds a correction for scale-dependence. These two forms are then combined to yield a four parameter function that we use to compute reference penetration in our laboratory. Parameters for these functions are computed for a compilation of laboratory scale data for tungsten rod penetrators fired against armor steel targets.

ANALYTICAL FORMULATION

It is well known that the normalized penetration (P/L) of rod projectiles as a function of impact velocity (v) follows an S-shaped curve. There is a threshold at which penetration begins. There is an inflection point beyond which the rate of increase of P/L decreases. Finally, there is a high velocity plateau, usually called the hydrodynamic limit.

The Lanz-Odermatt function was originally proposed in [1] as a semi-empirical function for the ballistic limit thickness of an oblique plate as a function of impact velocity, penetrator L/D, plate obliquity, and material properties. Four empirical parameters were used in this function, although for the case of high aspect ratio rods (L/D>20) impacting zero-obliquity plates only a single velocity scaling parameter was required. The upper limit of the function in this case was assumed to be the hydrodynamic limit derived from the ratio of densities.

The success of this form led to its use as a correlation for normalized penetration (P/L) into semi-infinite targets. Typically it is presented as a two parameter empirical equation:

$$P/L = A\exp[-(b/v)^2] \tag{1}$$

The parameter A represents the limiting value of normalized penetration at high velocity and depends on L/D and the ratio of densities. The parameter b is a characteristic velocity. These parameters may be related to mechanical properties of the penetrator and target as described in [1,2]. Our motive in modifying this widely used function is that it fails to accurately fit tungsten rod data over the entire velocity range of interest to our institute.

Generalized Lanz-Odermatt

Our first modification, which we call a Generalized L-O function, allows the velocity exponent to vary as a third parameter to the function:

$$g = A\exp[-(b/v)^c] \tag{2}$$

Here “g” represents normalized penetration (P/L). The primary advantage of this form is that the transition between the low velocity trends near the inflection point and the higher velocity plateau becomes adjustable. Fig. 1 shows a normalized form of this function in which *g/A* is plotted as a function of normalized velocity *v/b*. The curves have a common value *1/e* at a normalized velocity of unity with the exponent *c* acting to expand or compress this function about this point. This parameter therefore affects the slope in this steep region of the function. The slope at this common point is equal to *c/e*. However, it should be noted that the inflection point (i.e., the point of maximum slope) occurs at a lower normalized velocity that is dependent on the value of *c*.

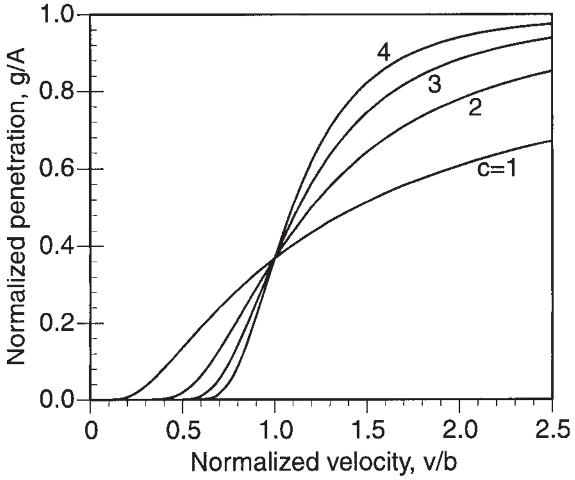

Figure 1. Generalized Lanz-Odermatt fit.

Scale-dependent Lanz-Odermatt

The existence of scale effects is well-establishod and can be significant [3]. For a fixed L/D penetrator, normalized penetration (P/L) is reduced at smaller test scales. Strain rate effects or flow stress have been shown to contribute to this effect but do not account for the observed reductions [4]. Other explanations include the scale of strain localization in the penetrator and/or target (which may be affected by target layering) and material sampling variation. To extend our reference correlations to a wider range of test scales, the scaling approach proposed by Me-Bar [5] was applied to the Lanz-Odermatt function.

Me-Bar's scaling methodology is an energy-based methodology. It is assumed that energy is dissipated by volume effects and surface effects. It is not important to define the specific processes involved, only to recognize that two classes of processes exist. Experimental data at two scales are used to deduce the relative contribution of these two classes.

The methodology assumes that the correlation of penetration and velocity is affected by the surface-volume ratio. Thus this model involves inserting a scale effect into the velocity-dependence term of eq. (2). In Me-Bar's application of his scaling methodology to semi-infinite penetration he compared the velocity required to achieve a normalized penetration depth (P/L) of unity with the required energy related to the square of this velocity. In a similar manner, the velocity parameter b in the Lanz-Odermatt function is the velocity required to achieve a normalized penetration depth ot A/e. Defining a characteristic velocity b_o, which represents that obtained in an infinite scale experiment (in Me-Bar's scaling methodology scale effects disappear as test scale approaches infinity), we obtain:

$$b = b_o(1 + K/D)^{1/2} \tag{3}$$

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Here K represents the characteristic length of the penetration process. The ratio K/D is related to the relative contribution of surface effects and volume effects. Both b_o and K are functions of the aspect ratio of the penetrator as well as the material properties. The argument is that as the diameter of the penetrator decreases the relative fraction of energy spent on surface effects increases. When D becomes less than K, these surface effects begin to dominate. Substituting into eq. (1) yields a scale dependent form of the L-O function:

$$g = A\exp[-(b_o / v)^2(1 + K / D)] \tag{4}$$

Scale-dependent Generalized Lanz-Odermatt

The forms suggested above each add a third parameter to the standard L-O function but for different reasons. Combining these two forms yields a four parameter fit:

$$g = A\exp[-(b_o/v)^c(1 + K / D)^{c/2}] \tag{5}$$

This function follows the same trends as the L-O function but provides greater flexibility in the transition to high velocity behavior and a scale dependence effect.

APPLICATION TO TUNGSTEN ALLOY PENETRATORS

Data were collected from a variety of sources [2,6,7,8] grouped into sets of common aspect ratio of approximately 20 and 30. Targets are armor steel with a nominal hardness of 270 BHN. Projectiles are tungsten alloy with densities ranging from 17.3 to 17.8 g/cc. Differences in target hardness and penetrator density were corrected using a method derived from the dependence of normalized penetration on velocity [9].

$$g_1 = g_0\left(1 + \frac{\Delta\rho}{\rho_0}\right) + \left(\frac{\partial g}{\partial v}\right)\frac{v}{2}\left(\frac{\Delta\rho}{\rho_0} - \frac{\Delta R_T}{R_0}\right) \tag{6}$$

The derivative of the function was taken from L-O fits to data of common aspect ratio.

The Generalized L-O function (2) was fit to sets of L/D 20 and 30 data. The resulting parameters are listed in Table 1. The L/D 20 correlation, which is based on the largest collection of data, is shown in Fig. 2 along with a L-O fit to the data (A=1.959, b=1.417 km/s). Systematic errors are present in both fits, particularly near the knee, but the greater flexibility provided by the third parameter of the Generalized L-O fit reduces overprediction of the data at hypervelocity. Better agreement could perhaps be obtained if additional hypervelocity data were included. Both fits are reasonably good for laboratory scale penetrators over a wide velocity range, but even within the small range of test scales considered (1/4 to 1/2, when full scale refers to rods 25 mm in diameter) some scale dependence is evident.

Both scale-dependent forms (4, 5) were fit to the data sets. The Scale-dependent L-O form (4) had the same tendency to overpredict at the higher velocities and seemed to show excessive scale dependence in the ordnance velocity range. Parameters obtained in the

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fits are given in Table 1. The fit of the four parameter Generalized L-O function (5) to the L/D 20 data is shown in Fig. 3, where it is plotted for several representative penetrator diameters. Systematic errors persist, but the function follows the expected trend of scale effects diminishing with impact velocity.

Table 1. Empirical parameters for tungsten long rods

	L/D 20			L/D 30		
Parameter	Eq. 2	Eq. 4	Eq. 5	Eq. 2	Eq. 4	Eq. 5
A	1.770	1.921	1.767	1.720	1.948	1.760
b (km/s)	1.357	1.165	1.147	1.522	1.196	1.186
c	2.497	-	2.403	2.912	-	2.489
K (mm)	-	3.533	3.066	-	4.872	4.037

OBSERVATIONS

In all fits in which the exponent for velocity dependence was allowed to vary (the parameter c in the eqs. 2 and 5), it exceeded the value of 2 assumed in the Lanz-Odermatt function. While it is convenient to consider the characteristic velocity *b* as related to the material strengths, the observed values of c would suggest a strength term that decreases with impact velocity, and hence strain rate, in contrast to data derived from material tests. A more likely explanation is that multiple processes are involved, for example deceleration of the penetrator at lower velocities or compressibility effects at the higher velocities.

The parameters given in Table 1 are for a least-squares best fit over the entire region. Other, slightly different values, will be desirable in some applications to obtain a best fit over a small velocity range. For example, in the region of greatest concern to our institute: 0.4 to 6MJ rods, L/D≥30, 1.8<v<2.6 km/s, can obtain a slightly better data fit with A=1.7, b=1.2 km/s, c=2.9, and K=3.5 mm.

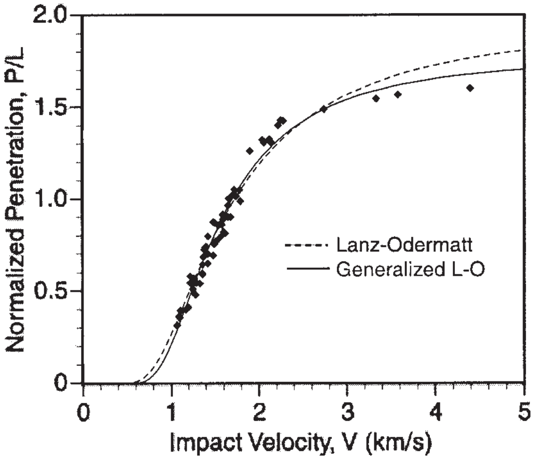

Figure 2. L-O and generalized L-O fits to L/D 20 data.

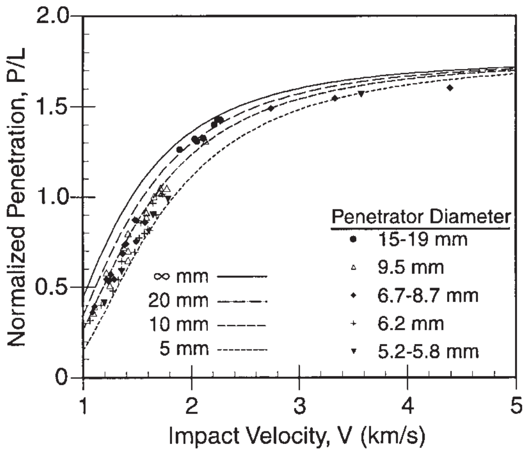

Figure 3. Scale-dependent GLO fit to L/D 20 data.

The results from the scale-dependent fits suggest several trends, although it is not clear whether these are real or artifacts of the assumed scale dependence. Recall that the scaling approach is based upon physical arguments that are then incorporated into an empirical model.

First, as observed by Me-Bar, the characteristic length K increases with aspect ratio of the penetrator. This implies that scale effects are more pronounced for higher L/D rods, at least when comparing rods of a given diameter. More importantly, the values observed for K are of the order of diameters used in laboratory-scale tests. This implies that care should be taken when extrapolating laboratory-scale results to “full-scale” predictions.

Secondly, the predictions suggest that scale effect persist at velocities extending into the hypervelocity range. This effect is shown in Fig. 4, in which P/L for several diameters of L/D 20 rods is normalized by that expected for an infinite diameter rod. The parameters were taken from the fit of eq. (5) to the L/D 20 data. The figure suggests that even at 2.5 km/s a laboratory-scale penetrator (~5 mm diameter) achieves only 90% of the normalized penetration that could be obtained with a sufficiently large diameter penetrator.

This second observation challenges the conventional wisdom that scale effects are minimal at these higher velocities. However, there are so little large-scale, hypervelocity data that the accuracy of the prediction cannot be assessed.

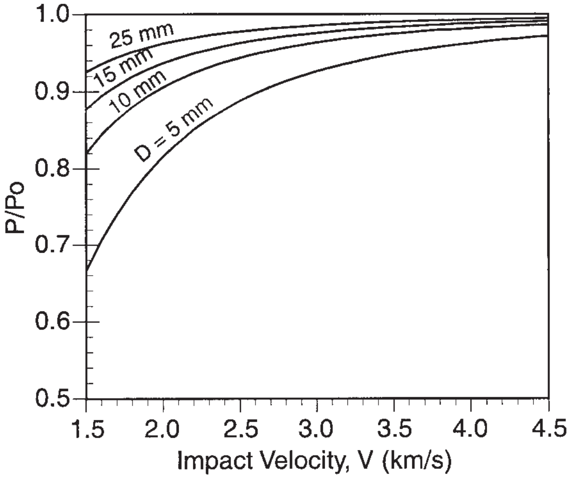

Figure 4. Diameter effect on L/D 20 penetration.

CONCLUSIONS

Several function forms, all based on the Lanz-Odermatt (L-O) equation, have been proposed for empirical correlations of semi-infinite penetration data. These equations were then fit to compiled data sets for tungsten rods striking steel targets.

Converting the exponent for velocity dependence into a parameter altered the transition from low-velocity to high velocity. For the data sets considered this exponent was consistent greater than the value of 2 assumed in the L-O equation. This causes a steeper approach towards the asymptotic value A and a sharper “knee” in the curve.

Scale-dependence was added using an energy-based empirical approach. Fits to the data sets indicate that scale effects are more significant for higher aspect ratio rods and suggest that scale-effects extend into the hypervelocity regime.

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