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FURTHER ANALYTICAL MODELLING OF SHAPED CHARGE JET BREAK-UP PHENOMENA

K. G. Cowan, B. Bourne

DERA, Fort Halstead, Sevenoaks, Kent, UK, TN14 7BP.

An analytical model has been developed from first principles in order to predict the break-up parameters of a shaped charge jet. The model is based upon the Goldthorpe analysis of tensile instabilities [9] coupled to the wave analysis of plastic perturbations on a stretching rod. A Zerilli-Armstrong constitutive equation for BCC metals was used. An analytical jet formation code (JET) and a hydrocode (GRIM), which were developed by DERA, were used in order to calculate the initial strain, strain rate and temperature in the jet. Various break-up parameters were calculated from the model, and in particular, the value of the plastic particle velocity (ΔV_{pl}) which represents the average velocity difference between broken particles. The predictions of the model were compared to the characteristics of a real jet composed of tungsten, which exhibited high dynamic ductility, and good agreement was obtained.

1. INTRODUCTION

The break-up of shaped charge jets has exercised the minds of several workers [1-8]. Some of the previous analysis, however, has been incomplete since it did not explain why such plastic perturbations should arise. This present analysis will address that problem and also it will utilise the more rigorous tensile instability conditions, which were developed by Goldthorpe [9]. The objective of this study is to generate a jet break-up model entirely from first principles based upon the application of a known material algorithm.

2. PERTURBATION MODELS

In the past, the majority of analytical break-up models were based upon the analysis of plastic perturbations on stretching metal rods. There was one fundamental criticism of this approach in that the predicted jet break-up would occur in relation to the positions of the perturbations. This is effectively a self-fulfilling prophecy and it does not explain how the initial perturbations arise. However, it has recently been demonstrated by Goldthorpe [10] that the presence of axial stress gradients along a stretching jet will give rise to a series of necks, provided that the material algorithm has the correct form (e.g. having a work hardening feature). For a BCC metal, the flow stress can be represented by the following simplified equation:

$$\sigma = k_1(d\varepsilon/dt)^m + k_2 + k_3\varepsilon^n \tag{1}$$

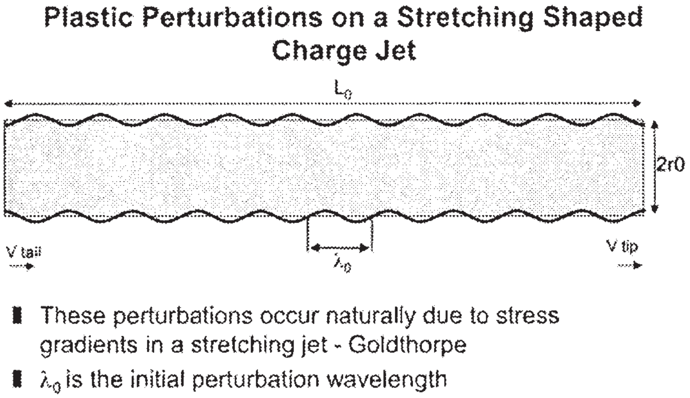
It may be shown that the rate of change of the jet radius, r, can be related to the variation in the flow stress.

$$-dr/dt = r/2[(\sigma - k_2 + 2nk_3 \ln(r/r_0))/k_1]^{1/m} \tag{2}$$

Consequently, a small variation in the flow stress, σ, will lead to a variation in the radius of the jet as a function of time and hence the formation of a series of necks. These variations in flow stress along the jet have been observed in the hydrocode analysis of the problem (Figure 6) and hence confirm that the conditions exist for the development of tensile instabilities. This analysis produces a result that can be related to the perturbation approach, which has been adopted by others. In Goldthorpe's analysis the axial stress gradient is raised to a large power and fairly small stress gradients can consequently lead to the formation of tensile instabilities and hence necks in the material. Hence the perturbation analysis is now demonstrated to be valid and has a more sound theoretical footing. Another potential source of the perturbations is that caused by the presence of hydrodynamic reverberations (elastic waves) at the onset of jet formation, although under certain conditions these may dissipate naturally. The model that we have chosen to develop is that of Walsh [2] where he relates the wavelength of maximum disturbance growth to the initial jet radius. In Walsh's original paper he also suggests that variations in yield strength can lead to the development of perturbations along the jet.

$$\lambda_i = \pi r_0 \tag{3}$$

Figure 1.



Plastic Perturbations on a Stretching Shaped Charge Jet

- These perturbations occur naturally due to stress gradients in a stretching jet - Goldthorpe
- λ_0 is the initial perturbation wavelength

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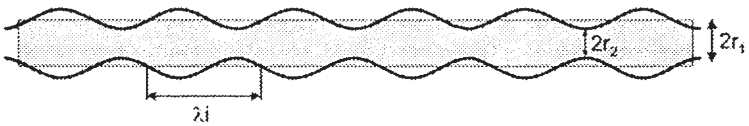
Figure 2.

Plastic Perturbations on a Stretching Shaped Charge Jet Just Prior to Fracture



In Figures 1 and 2 the initial disturbance wavelength, λ_0 and the final disturbance wavelength, λ_i, are shown. There are N standing waves along the stretching jet initially and they expand as the jet stretches [4,5]. From the equation of motion of a jet particle the individual break-up time may be defined by:

$$t_b \Delta V_{pl} = \lambda_i - \lambda_0 \qquad (4)$$

where ΔV_{pl} is the velocity difference along an individual jet particle.

This leads to the final expression for break-up time:

$$t_b = \pi r_0 / \Delta V_{pl} - (dt/d\varepsilon)_0 \qquad (5)$$

where $(d\varepsilon/dt)_0$ is the initial strain rate in a stretching jet.

From Figure 2 the strain at necking is defined by:

$$\varepsilon_N = \ln(A_1/A_2) = 2\ln(r_1/r_2) \qquad (6)$$

3. MATERIAL ALGORITHM

The material model, which was used was the Zerilli-Armstrong form for BCC metals and the flow stress is then given by:

$$\sigma = C_1 \exp[-C_2 T + C_3 T \ln(d\varepsilon/dt)] + C_4 + C_5 d^{-1/2} + C_6 \varepsilon^n \qquad (7)$$

The values of the constants, which were used in the material algorithm are those obtained from the literature for re-crystallised tungsten [11]. The condition for tensile instability is the one, which was derived by Goldthorpe [9] based upon the appropriate form of the material algorithm and it is not the maximum load condition. The analysis by Goldthorpe is based upon minimising the radius of curvature of the necked region and from that a set of tensile instability conditions has been developed. A detailed analysis of this criterion is beyond the scope of this paper but a full account of this is given at [9]. For BCC metals the instability condition becomes:

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$$0 = [\,1-3C_3T]\,C_1\exp[-C_2T+C_3T\ln(d\varepsilon/dt)]+C_4+C_5d^{-1/2}+C_6\varepsilon_N^{n}-nC_6\varepsilon_N^{n-1} \qquad (8)$$

This equation may be solved to give ε_N the strain at necking. In order to calculate ε_N one must have the values of the temperature and the strain rate ($d\varepsilon/dt$). To do this it is necessary to carry out a hydrocode simulation using GRIM. The strain rate and internal energy can be extracted at a number of positions along the jet. The temperature is calculated by using the polynomial form for the specific heat capacity.

4. THE ANALYTICAL BREAK-UP MODEL

This model applies to the ductile fracture of jets based upon the formation of multiple necks, which lead to tensile failure. The analytical break-up model is based upon the premise that the maximum velocity difference along a jet element is dependent on the magnitude of the perturbation (in terms of jet radii) and hence the strain at necking. The plastic particle velocity is defined as the maximum velocity difference a stretching rod can maintain before fracture occurs. When the magnitude of the perturbation produces a strain which is equal to that of the necking strain then fracture will occur.

The plastic particle velocity is also defined by:

$$\Delta V_{pl}=\int[(d\sigma/d\varepsilon)/\rho_0]^{1/2}d\varepsilon \qquad (9)$$

The plastic particle velocity may also be related to the strain at necking, by use of the material algorithm for BCC metals, by the following equation:

$$\Delta V_{pl} = 2/(n+1)\,(nC_6/\rho_0)^{1/2}\varepsilon_N^{(n+1)/2} \qquad (10)$$

Since the strain rate and temperature will vary along the jet, the necking strain and hence the plastic particle velocity may also vary. From the hydrocode analysis the strain rate and temperature were calculated for 20 positions within the jet and the necking strain and hence the plastic particle velocity could be calculated from equations (8) and (10). In Figure 3, there is a plot of the predicted plastic particle velocity as a function of jet velocity for a ductile tungsten jet and this is compared to the mean experimental value. The predicted value of the plastic particle velocity is also compared to the individual empirical values in Figure 4. In Figure 5, the experimental jet particle lengths are compared to the predicted particle lengths based upon the break-up model. The number of jet particles is given by:

$$N-1 = (V_{tip}-V_{tail})/\Delta V_{pl}) \qquad (11)$$

If the plastic particle velocity varies along the jet then one can introduce the concept of an average velocity in order to calculate the number of particles. Hence the average value of the plastic particle velocity is defined by:

$$<\Delta V_{pl}>(\varepsilon_2-\varepsilon_1) = \int \Delta V_{pl} d\varepsilon \tag{12}$$

Alternatively, the number of jet particles may be calculated from the following equation:

$$V_{tip}–V_{tail} = \int \Delta V_{pl} dN \; [1,N] \tag{13}$$

This analytical model can be applied sequentially along the jet provided that the strain rate and temperature are known. It is also possible to calculate the initial strain rate from the output of a jet formation model such as DERA’s JETFORM. Alternatively the strain rate can also be extracted from the output of a DERA hydrocode such as GRIM at various stations along the jet.

5. ANALYTICAL JET FORMATION MODELLING

The initial strain and strain rate in a stretching shaped charge jet can be extracted from the output of an analytical model such as JETFORM. The definition of trne strain, s, is given by:

$$\varepsilon = \ln(l/l_0) = \ln(l/\Delta V \Delta t), \tag{14}$$

where ΔV is the jet velocity difference between time-steps and Δt is the time-step.

The strain rate, dε/dt is then given by:

$$d\varepsilon/dt = 1/l \; dl/dt = \Delta V/l \tag{15}$$

These parameters can be used as input to the material model in order to calculate the flow stress along the jet.

6. HYDROCODE MODELLING

A full hydrocode run using a two dimensional Eulerian axi-symmetric hydrocode, GRIM was used to simulate the problem. A Mie-Gruneisen thermodynamic equation of state was used as well as a Zerilli-Armstrong material algorithm for re-crystallised tungsten. It was possible to extract the strain, strain rate and internal energy and hence the temperature at a number of positions along the jet. These parameters can then be used as input to the instability model in order to calculate the values of the plastic particle velocity and the variation of the flow stress along the jet.

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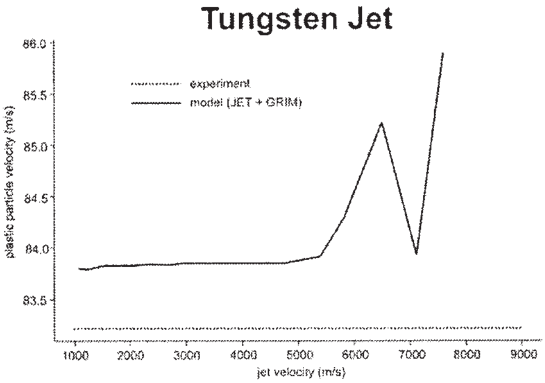
7. RESULTS

The predicted value of the plastic particle velocity was typically 83 m/s along most of the jet and rose to about 86 m/s near the jet tip (Figure 3). The mean experimental value was 83.2 m/s, whilst the individual values were scattered over a range from l50 m/s to about 30 m/s as shown in Figure 4. The individual jet particle lengths are shown in Figure 5 and there was reasonable agreement between the experimental values and the prediction of the model, although there is some scatter, which will be discussed later. The strain at necking does not vary very much along the jet and is of the order of 0.3. This may help to explain why the plastic particle velocity is virtually constant along the jet and why the semi-empirical analytical jet break-up models work so well. This implies that that ratio of the necked radius to that of the unperturbed radius is 0.86.

Figure 3.



8. DISCUSSION

This study has uncovered several interesting features that relate to shaped charge jet break-up phenomena. The fact that the magnitude of the plastic particle velocity can be predicted very well by the use of a hydrocode and a suitable material model implies that this technique can be used as a predictive tool for the determination of shaped charge jet break-up parameters. The form of the material algorithm is well established for BCC

Figure 4.

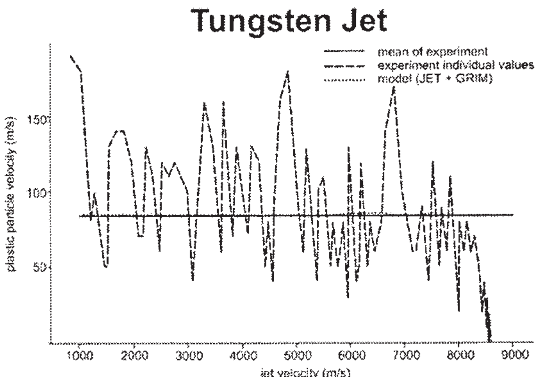


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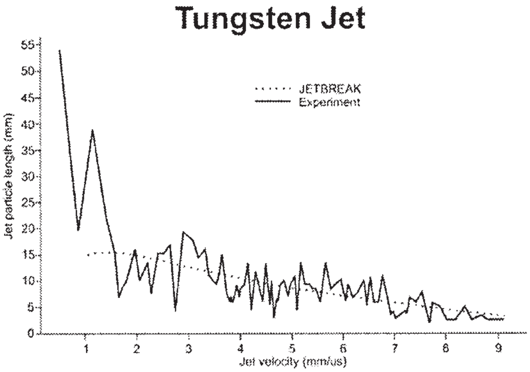
Figure 5.



metals and it seems to work well in this case. For FCC metals, new, path dependent algorithms have been developed [12] and they need to be tested using this model. Another significant feature of this topic was the degree of randomness, which was associated with the break-up phenomena. Whilst the mean of the experimental plastic particle velocity was very close to that of the predictions of the model, the spread of values was significant. This implies that there is some underlying stochastic process, which relates to the jet break phenomena. At one level the break-up of the jet is deterministic but there are stochastic effects which are superimposed upon it. The magnitudes of the jet particle lengths were also random although they followed a general trend in relation to the theoretical model. The wavelength for maximum disturbance growth is not a unique entity and there will be harmonics and sub-harmonics of this value. In addition to that the value of the critical wavelength as determined by Chou and Walsh [1,2] is given by:

$$\lambda_c = \pi r_0/\sqrt{2} \qquad (16)$$

Certain parts of the jet may break earlier than that predicted by the model due to these stochastic effects and other parts of the jet may contain multiple wavelengths which are "frozen in" due to inertial effects. All of this requires further investigation before we have a full understanding of jet break-up behaviour. Other features of jet break-up become apparent when one considers the relevant equations and in particular equation (5). The smaller the value of the plastic particle velocity, the longer is the break-up time and the greater is the cumulative jet length. The number of jet particles, in the same velocity interval (V_{tip}–V_{tail}), also increases as the plastic particle velocity decreases. These models only apply to ductile fracture and they will break down if non-ductile, brittle or shear fracture occurs.

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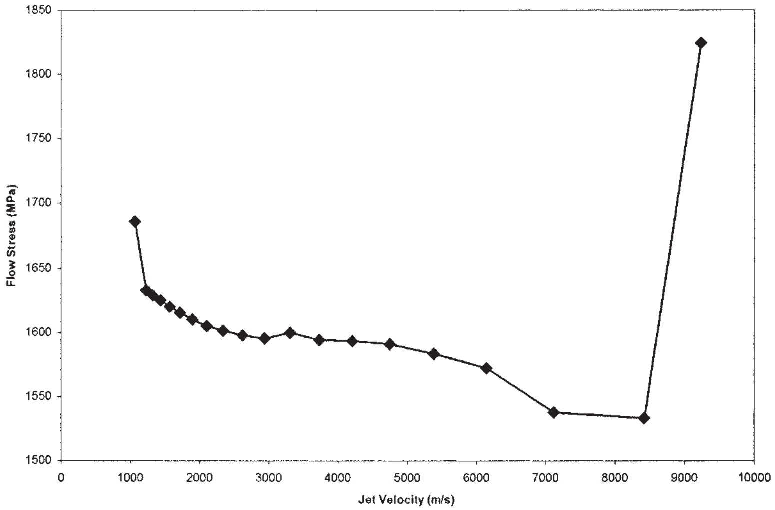
Figure 6 . The variation in flow stress with jet velocity for a tungsten jet.



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