THE INDETERMINACY OF THE OUTGOING FLOW OF TWO IMPINGING ASYMMETRIC JETS

Dr. Simcha Miller, Mr. Roy Ceder

RAFAEL Ballistic Center, P. O. Box 2250 (M1) Haifa, ISRAEL

In this work we show the difference between, the existence of rules governing the phenomenological as well as the computational aspects of collapsing symmetrical liners or shells and jet formation, and the absence of such rules in the asymmetrical case. Based on analytical work in the field of potential flow, it was clear that a single equilibrium solution for the outflow of impinging jets or shells does not exist. Based on detailed numerical simulations of realistic asymmetrical impinging shells we come exactly to the same conclusion as with the analytic potential flow. The simulations show clearly that any solution depends on the details of boundary and initial conditions, and not on the existence of a single equilibrium stable solution, based on a general variational principle. The significance of those conclusions are crucial when dealing with asymmetrical flows because of the absence of any simplifications like quasi-steady Bernoulli type modeling, which has been used extensively in the theory, as well as design work in the field of shaped charges.

INTRODUCTION

For many years the analysis of the jetting phenomena was based primarily on the quasi steady state assumption of the flow near the jetting region, of a collapsing axi-symmetrical liner, or a symmetric linear liner collapse. The whole jetting process was demonstrated by detailed numerical simulation, however, most design tools, analytical, and even simulational, were strongly based on a quasi-steady flow assumption. In all the simulations and models, the symmetry of the process was not just implied, it was the only one condition imposed on a complete numerical simulation, or on a Bernoulli type solution.

In an analogous way recent works tried the same procedure to develop the foundation and to solve the problem of the jetting from asymmetric liner collapse [1-5]. There are also works concerned with the influence of slight asymmetries, within the allowable tolerances, on the performance of precision shaped charges [6-8]. In this work we are mainly concerned with full scale asymmetries. Surveying the literature on this problem, we inevitably arrive at the conclusion, that the same type of a general procedure, resulting in fairly quantitative comparison to all known experimental results does not exist. Even qualitative results are far from satisfying. They fit poorly the experimental results. If they tend to resemble locally along part of a jet, they fall far away in other parts.
In this work we try to establish the indeterminacy of the solution of the outgoing flow of two asymmetric impinging jets. This was actually the point of view formulated in the fifties for potential flow [9]. It is worthwhile to quote two sentences formulated by Birkhoff in [9]. “Physically, it is natural to suppose that if the incoming jets are given, then the resulting flow in determined. However it can be shown that, contrary to expectation, the resulting flow is in fact indeterminate, except in the case of parallel impinging jets!” and the second sentence states: “The Physical significance of this indeterminacy is hard to grasp; all flows are in equilibrium. It is not clear what is the condition, if any, for the stable equilibrium of non-parallel impinging jets. It may be that all stationary configurations are unstable”. Birkhof’s statements does not rule out the possible existence of a variational principle which should be imposed on the flow, it does not rule out also, the indeterminacy, in the symmetric case if the symmetry is not imposed on the flow.

Birkhoff’s arguments are absolutely correct for potential flow without imposing any additional constraints. One might assume that in the case of a realistic flow, including well defined material like copper for instance, with reasonable equation of state, the results of detailed simulation of the out going flow will reveal the existence of general principles leading to the unique solution which is the stable one. We will show in detail that this is not the case, and that the indeterminacy is true in general for any flow independent of its material properties. Using the Eulerian processor of the AUTODYN, we will show how easily we can switch between highly different solutions for exactly the same problem, or actually determine arbitrarily the solution as we wish. The only conditions that all solutions should fulfil is, of course, the mass and momentum conservation between the incoming and outgoing flows.

The most striking result is that even the solution in a symmetric case is not a unique one, or the most stable one. If the symmetry is not imposed on the analysis or the numerical configuration, we can easily derive infinite different solutions, just the same as in the asymmetric case, by slightly manipulating the initial conditions of the impinging two similar streams. Once, an asymmetrical solution is established, any further continuation of the simulation shows no sign of shifting the solution towards the symmetric one.

In the next 4 sections we describe the following processes.

- The strong dependence of the resulting out flow on the variation of the EOS (Equation of State).
- The independence of the resulting out flow on the EOS if the flow has already been established by either different EOS, or by different initial conditions.
- The strong dependence of the resulting out flow by changing the initial condition.
- The strong dependence of the resulting out flow on initial conditions in the symmetric case.

In the last chapter we add a comment on parallel impinging jets.
VARIATION OF EOS

The flow configuration we chose to demonstrate the indeterminacy consist of: Two plane streams having the same speed (3 mm/ms) are moving into each other with an angle of 120° between them. The width of the upper flow moving downward is 0.4 times the width of the lower flow moving upward, see Figure 1. The incoming flows are arranged to move exactly along the direction of the grid lines, I lines in the AUTODYN, filling exactly the interval between them. The direction of the I lines change by 60° at the central subgrids. The J lines are kept horizontally in both the upper and lower planes. In this arrangement the incoming flows move smoothly from the boundaries, where we apply the free flow conditions down to collision area. To keep a good level of accuracy, we use a high grid density around the central subgrids where the collision and the build up of the amount and direction of the outflow take place. The details of the grid are shown in Figure 1 and explained later in the discussion section.

Fig. 1: The initial grid setup. The upper left corner is shown magnified on the right side.

The difference in EOS

Figures 2 and 3 demonstrate the simulational results of exactly the same initial flows, but with a single (albeit significant) difference in one parameter in the EOS. We use the hydro option (of otherwise the full Steinberg-Guinan model) of the AUTODYN changing only the parameter S in the relation:

\[ U_s = C_0 + SU_p \]  \hspace{1cm} (1)
Where $U_s$ is the shock velocity, $U_p$ the particle velocity and $S$ is coefficient adjusted for each material. By hydro we mean that the material has no strength, the flow stress and the shear modulus are set to zero. First we simulate the flow with the nominal value of $S = 1.489$. Then we vary the parameter significantly to $S = 5.$ and repeat the simulation from the beginning. The slope of any isentrope in the $(P,V)$ plane and especially in the hugoniot changes dramatically with increasing $S$. Increasing $S$ simply means moving towards incompressible fluid. For instance, the limiting jump in density is given by:

$$\frac{\rho_s}{\rho_o} = \frac{S}{S-1}$$  \hspace{1cm} (2) \hspace{1cm} \text{Where $\rho_s$ is the shocked density, and $\rho_o$ is the initial one.}

Fig. 2: The resulting outflow in the simulation with the nominal $S=1.489$. Fig. 3: The resulting outflow in the simulation with $S=5$.

**Results**

Figure 2 demonstrates the outflows. The thicker one moves to the right in a direction slightly above the horizontal, while the thinner one takes a direction which creates an angle of about $38.88^\circ$ to the upper incoming stream. The horizontal direction has an angle of $60^\circ$ to the upper initial stream.

In Figure 3 we show exactly the same initial flows, but in this case the value of $S$ was chosen to be much higher $S = 5$. The resulting outflows are significantly different. The thinner outflow moves along a direction close to the horizontal one. The angle between the thinner outflow to the upper inflow is about $54.58^\circ$, a change of about $16^\circ$, compared to the direction with the nominal value of $S$.

**EQUILIBRRUIM**

Now we turn to the question, whether the two solutions for the nominal EOS and the one with $S=5$ are the unique equilibrium solution? To answer this question, we use the two stable solutions as initial conditions for the next stage, where we switch the value of the parameter $S$ between them. Figure 4 shows the results of the continuation of the simulation of the flow in Figure 2 after changing the value of $S$ from the nominal to the value of 5. Figure 5 shows the results of Figure 3 after switching the value of $S$ from 5 to the nomi-
nal 1.489. Practically there is no change between Figure 2 and Figure 4, and only a slight change in angle between Figures 3 and 5 of about 1.2° towards the horizontal direction. That slight change is insignificant compared to the change of about 16° between the two solutions, and actually it is also in the wrong direction! The results after the switching values of S, clearly, show that all solutions are equally stable once they are formed, and there is no preferred stable solution depending on the EOS. Before getting to the heart of the problem, namely, the indeterminacy even in the symmetric case, we show in the next section how we can change final flows by manipulating temporarily the initial conditions, and not just by different EOS.

Fig. 4. The result of the continuation of The solution in Fig. 2, obtained with S=1.489 after switching to S=5.

Fig. 5. The result of the continuation of the solution in Fig. 3 obtained after switching to S=5.

### VARIATION OF INITIAL CONDITIONS

Let us take again the solution established in Figure 3, with S=5, and start its simulation again with a slight change in initial conditions. The upper initial inflow is filled down to the central area. The lower initial inflow fills the lower part up to a gap of 1.09 mm to the central area. In this case the initial condition takes place slightly below the collision plane. This change, by no means, changes anything significant in the problem. For given two incoming streams with well defined width’s and center lines, the collision plane is defined uniquely, and if a single stable solution existed, the slight change in initial condition can only delay slightly the appearance of the unique stable solution, but it cannot change it to a completely different one.

In Figure 6 we show the results of that simulation. The out flow in this case is much closer to the results shown in Figure 2 with S=1.489, the nominal one. The angle with the initial inflow is even slightly smaller by about 1.3°. As we see, we can achieve about the same effect either by a significant change in EOS, or by a slight change initially in the location of the collision. We are going to use that mechanism to test variations in the symmetric case.
THE SYMMETRIC CASE

In this case we take the same flow configurations with the upper incoming stream equal in width to the lower thicker one in the previous cases. The angle between the two streams is the same 120°. In Figure 7 we show the symmetric solution. To increase the initial disturbance, the initial gap between the front of the lower incoming stream and the central area was set to almost 11 mm. In that case, when the collision begins, the upper stream has already reached the lower grid boundary. Any further increase of the gap will result in the same flow. It just delays the beginning of the collision at exactly the same point with exactly the same initial conditions. Figure 8 shows the striking result, which consists of two non symmetric streams, producing different angles with the central collision plane. The angle between the thinner outflow and the upper incoming stream is around 51.7°. The flow in Figure 8 is kept in the same position, without any observable change for more than 40 µs.

DISCUSSION

For the sake of consistency we kept the same grid shape and density in the central subgrids, where the significant interaction takes place, and the same logarithmic increase in the outer subgrids, as shown in Figure 1. Likewise we kept the same angle between incoming streams, and their widths. In the case of the symmetric problem both streams share the same width as in the thicker one in the previous cases. To maintain high numerical precision the central part divides the thicker (3 mm) incoming flow into 60 division. As a consequence, the thinnest outgoing flow is divided into at least 15 divisions. The saw teeth amplitude along the thinner outflow tend to increase as the flow moves from the central dense grid to the outer grid, where the spacing between divisions grow logarithmi-
The result of the symmetric flow where the collision starts symmetrically. Symmetric case with initial 10.93 mm gap below the collision plane.

In our simulations we found no dependence on strength. This is probably due to the high collision velocity in our problems, that produce near the stagnation points pressures, which are orders of magnitude higher than the strength. We chose intentionally $120^\circ$ between the incoming streams, to compromise between the requirements to have the whole incoming stream bounded exactly between two I lines, and to have, on the other hand, a grid shape which is not too far from a square. However, we took care to have a very dense square grid in the central part. In our study we manipulated the initial conditions by delaying slightly the collision between the two incoming flows. There are other powerful methods, for instance, starting with different initial velocities which gradually approach the final common velocity.

In the case of parallel impinging jets, potential flow predicts a single unique solution. We have grounds to believe that this is the case also in real flow. However, it cannot be proved by few simulations that show no contradiction. It needs further work to prove and substantiate it.

CONCLUSIONS

The most significant conclusion of this work is the clear observation of the difference between symmetric and asymmetric flow configurations. In any case the exact boundary and initial conditions, determine uniquely the resulting flow, and not a variational principle resulting in a single flow. Whereas in symmetric configurations, the flow towards the jetting zone, is practically symmetric, hence also the solution, the asymmetrical solution depends completely on the details of the incoming flows, and cannot be approximated or modeled, in a general way by a Bernulli type of flow, or any other type of approximation. The only way, to solve asymmetrical problems is the direct simulation which takes into account the exact boundary and initial conditions of the incoming flow fields, to produce the exact and uniquely defined outflow. It is speculated that even problems with small deviations from symmetry, suffer from the lack of a general solutions, and depend also on the details of the asymmetric flow into the jetting region.
REFERENCES